Stealing authority: A model of institutional transformation*

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Abstract

Fundamental political change often occurs gradually. Yet mechanisms of incremental institutional transformation are poorly understood. This paper presents a game in which players are entirely strategic and share proposal power, while showing how one player can gradually accumulate authority over time at the expense of another. The model analyzes a bargaining problem between two actors. Over an infinite number of periods, each actor has a chance each period to propose a bundle of future authority plus present policy, which creates the potential for actors to trade authority for temporary policy concessions. With some degree of noise in the division of authority, a proposal that transfers authority reduces the reservation value of the respondent, creating an incentive for the proposer to trade present policy for future authority. The model provides a mechanism for how ongoing, everyday political interactions can gradually rearrange authority relationships among political institutions.

Keywords: endogenous institutional change, bargaining model, endogenous status quo, authority, coercive power

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Fundamental political change often occurs gradually. Examples include the rise of judicial review by the US Supreme Court, the growth of authority over national governments by the European Court of Justice (ECJ), the relationship between the US federal government and state governments, the relationship between legislative and executive authority in presidential democracies, and the authority of legislatures within many contemporary and historical authoritarian regimes, such as the British Empire. Sometimes political change is rapid, of course, but even then visible breaks often occur within the context of a transformed institutional terrain that developed slowly if not imperceptibly. For example, the American revolution only occurred after colonial legislatures had already accumulated substantial authority at the expense of the British crown.

How are we to understand such cases of gradual institutional transformation? A prominent approach in formal models of political development is to show how different equilibria depend on particular combinations of exogenous parameters, and then explain historical change by the change in exogenous parameters moving the outcome of the game from one equilibrium to another.[1] In this way, models may exhibit gradual institutional change through gradual movement of exogenous parameters. While this approach can be fruitful for many applications, it leaves the fundamental causes of change outside the model.[2] There is no sense in which the existing arrangement of institutions may imply the gradual transformation of institutional forms and relationships.

This paper presents a model in which one actor gradually accumulates authority at the expense of another. The model analyzes a bargaining problem between two actors. Each actor represents an institution, such as a federal government versus a province, or an

1. To the author's knowledge, Calvert (1995) provides the clearest description of this approach to studying institutional development.

2. In models in which there is a transfer of authority, such as models of suffrage expansion, if such a transfer of authority occurs it occurs immediately (though for an exception see Jack and Lagunoff 2006). Contrast this perspective to the gradual growth of Parliamentary authority or the decline of the House of Lords in Britain (used as an example in Mahoney and Thelen 2009).

executive versus a legislature. Over an infinite number of periods, each actor has a chance to propose a bundle of future authority plus present policy, which creates the potential for actors to exchange policy for authority. Crucially, there is some degree of uncertainty in the division of authority. In the model, a proposer's claim of authority might provide precedent in future bargaining, even if the respondent rejects the offer. This may be unlikely, but an arbitrarily small probability that this occurs creates the effect that, in some circumstances, the proposer now has an incentive to accumulate authority while providing present policy benefits to the respondent. Over many periods, policymaking authority is gradually transferred from one actor to the other.

Related literature

The model that I present is distinct from most models of gradual political change because of the focus on authority, rather than policy change or power (conceived of as a probability of winning in a conflict). An exception is provided by Howell et al. (2019), who analyze the gradual accumulation of authority by an executive in competition with the judiciary. In the model of Howell and colleagues, the president always has proposal power, and if the judiciary ever overrules a presidential action, then the judiciary can never approve a similar action in the future. However, Howell and colleagues assume that actors seek authority, whereas in the current model actors care about policy and only seek authority to that end. By incorporating both policy and authority into a bargaining model, we can investigate how actors trade off either option.

The important feature of this model is the allocation of authority across institutions. As conceived in this paper, an actor's authority guarantees a partial policy benefit in the absence of a successful bargain. Hence, moving the division of authority alters the reversion point for bargaining. Bargaining models with an endogenous status quo have analyzed legislative bargaining (Diermeier and Fong 2011, Kalandrakis 2010, Nunnari 2019), and have incorporated changing preferences (Dziuda and Loeper 2016) and choices between distributive and public-goods policies (Battaglini and Coate 2007). In contrast to these models, the current model focuses on bargaining over authority, which allows an actor to set policy unilaterally up to some limit, rather than bargaining over policy alone.

Likewise, bargaining over authority is distinct from models in which actors bargain over power, conceptualized as the probability that an actor wins in a fight. Fearon (1996) presents a model in which actors trade portions of a pie, which then implies a probability of winning a potential conflict. Fearon shows that if the potential probabilities of victory that are subject to bargaining are continuous, then no conflict occurs. In effect, the proposer is able to use their proposal power in order to extract concessions from the respondent, due to the inefficiency of conflict. The two most important differences between my model and Fearon's is that, in my model, actors trade authority rather than a probability of winning a conflict (and present policy and authority move independently), and the role of coercive power in addition to proposal power in driving the outcome of the game.

Coercive power occurs when one actor has the ability to reduce the reservation value of another, affecting which bargains the second actor will accept and benefiting the first actor (Acemoglu and Wolitzky 2011, Chwe 1990, Dal Bó et al. 2006). Consider as an example the "bargain" between a mugger and a pedestrian. In the absence of coercion, the pedestrian would not hand over their wallet ("your money or nothing"), but the mugger's gun reduces the pedestrian's reservation value ("your money or your life"), and consequently the pedestrian accepts the mugger's offer. Coercion in this sense has been introduced in bargaining games to produce a gradual endogenous change in power, most notably by Powell (2013).

In Powell's (2013) model, a government gradually consolidates power while buying off a rebel group. Two actors representing a government and a rebel group bargain over a stream of pies for an infinite number of periods. The government can make an offer that both affects the probability of winning a fight in the future and the allocation of today's pie. Coercive power creates an incentive for the government to increase their probability of winning a fight, since that reduces the reservation value of the rebel group. Power is transferred to the government's benefit as the rebel group accepts pie today in exchange for a future reduction in military power.

In the current model, proposal power is shared by both actors through a random draw of who holds proposal power in each period. This provides, a “fair shot” for both sides to accrue authority. While in many contexts it may make sense for one actor to be able to use coercive power to disadvantage another actor, in many other contexts it does not make sense to assume ex ante that one actor or the other ought to be able to make use of coercive power. It is something to be explained why a particular institution is able to accumulate authority at the expense of another, when it would appear that both institutions have opportunities to propose offers that could move authority to their own advantage.

Moreover, the current model shows how relationships of authority can produce coercive power due to uncertainty over boundaries of authority. Because of this uncertainty, one player can reduce the reservation value of the other by offering current policy for future authority. This connects the concept of coercive power to existing conceptions of authority and also expands its application beyond contexts of explicit violence such as civil wars, corruption, or labor coercion. The model shows that relationships of authority can be transformed even in a context in which formal authority is binding, and all actors are constrained from engaging in explicit violence.

Summary of theory

Consider a stylized example in which two players divide a pie. Each day they receive a new pie to divide. There is some share of the pie over which each player has *authority*. This means that one has a right to a defined portion of the pie, and the other has a right to the rest. Each day, there is some probability that either player has proposal power—the ability to make an offer to the other that the other either accepts or rejects. If no bargain is reached, each player takes their rightful portion. However, it is possible for one player to exchange their portion of the pie for more authority in the future, so that one player gradually establishes authority over the entire pie, fully eliminating the authority of the other player.[3]

Because players care about policy, rather than authority directly, if this were all, players would be indifferent between trading policy now for policy in the future (properly discounted). However, an incentive to trade policy for authority arises from how the boundaries of authority are enforced. In addition to the two players bargaining, an audience (such as the citizenry) watches them imperfectly. The audience enforces the boundaries of authority between the first two players, and uses whatever division of authority was last agreed upon by both. In other words, when both players agree on a scope of authority, then this creates a "precedent" so that such a scope of authority may be defended in the future.

When one player offers the other a bundle of policy for authority, uncertainty over the boundary of authority can create an incentive for the proposer to move authority now and then take policy later. Because of this uncertainty, the enforcement of bounds of authority is imperfect. This means that, by offering a bundle that incrementally moves authority, a player can reduce the reservation value of their bargaining partner—the respondent might reject the offer but authority moves nonetheless.

Assuming a continuous bargaining space and sufficiently high cost that a player will not make an offer that they know will be rejected, the model shows three conditions necessary for a transfer of authority to occur:

- Some degree of uncertainty in whether or not violations of precedent will be punished.

  For instance, one way this could be interpreted is that, if a revision of authority re-

3. This example is inspired by James Harrington's example of two schoolgirls dividing a cake, which Harrington uses to illustrate how a separation of powers system could work to align public and private incentives. Harrington assumes that relationships of formal authority are followed.

lationships is rejected, then potential protesters may not notice or may otherwise fail to act collectively in order to reject the usurpation. So the claim to authority by one institution might become accepted in the future.

- Sufficiently biased proposal power toward one player is necessary for that player to have an incentive to give up policy gains now, and for the other player to accept policy gains rather than seeking to accumulate authority in their own favor.
- A sufficiently high discount factor, specifically for the player accumulating authority, is necessary for that player to be willing to give up present policy benefits. Importantly, players have a common discount factor, so the model shows that a high discount factor does not conflict with a player allowing authority to be withdrawn from them.

This stylized example reflects real-world bargaining over policy and authority between institutions. Consider the standard interpretation of the US Supreme Court case Marbury v. Madison, in which the Supreme Court established a lasting precedent for Court power of constitutional review in exchange for providing short-term policy benefits to the Jefferson administration, which accepted the decision. Alter (2001) argues that the ECJ established authority over national court systems in an analogous way, by issuing decisions granting policy wins to national courts while establishing ECJ authority over national court decision-making.

Another example of policy-authority bargaining is provided by colonial American legislatures within the English/British Empire in the 17th and 18th centuries. While founded sometimes without crown authorization at all or at least with little authority recognized by imperial officials, colonial assemblies gradually accumulated authority over numerous policy domains. Assemblies often used their budgetary power in exchange for further grants of authority by imperial officials (Greene 1963). After presenting the model, I will use this case to illustrate key elements of the strategic situation that the model elucidates.

The model

Consider an infinite-horizon game of complete information. There are two players, A and B. Each period, A and B have a pie of size 1 to divide. Both players' utilities are linear and increasing in pie. There is a common discount factor δ.

Bargaining

Either player has some probability of being the proposer in a period. Player A is the proposer with probability ρ, while player B is the proposer with probability $1 - \rho$. When a player is not the proposer, they are the respondent.

The proposer in a period offers an allocation of authority $x_{t+1} \in [0, 1]$ and a policy location $s_t \in [0, 1]$. A division of x_t results in player A obtaining authority over x_t portion and player B obtaining authority over $(1 - x_t)$ portion. This is the share of the pie that either player obtains in the future if no bargain is reached. Additionally, $s_t = 0$ means that B receives the whole pie, while $s_t = 1$ means that A receives the whole pie. This allows for the possibility of an exchange of future authority for current policy benefits.

The proposer chooses a tuple (x_{t+1}, s_t) that the respondent can accept or reject. Upon acceptance, then the pie in that period is allocated according to the offer, and the proposed division of authority then becomes the division of authority in the next period. Upon rejection, each player obtains their current policy benefits according to the existing division of authority. However, there is a probability ϵ that the proposed scope of authority in the following period, if it is different from that of the current period, will be successfully rejected. With probability $1-\epsilon$, the proposed scope of authority in the next period will prevail despite rejection by the respondent.

The share s_t represents current policy. A proposer can choose not to alter the existing policy (s_{t-1}) that they inherit. For instance, if the proposer in the previous period chose

$s = \tilde{s}$ and it was accepted, then assuming the other player holds proposal power in the following period, they could offer $\tilde{s}$ and have it be accepted for sure without needing to offer a shift in authority in exchange. This represents the proposer deciding to not move policy. In a given period, the proposer can always decide to leave policy at the status quo, which is the policy of the previous period.

Order of moves

The order of moves in a period is as follows:

1. Nature selects A to be proposer with probability ρ or B to be proposer with probability $1 - \rho$.

2. The proposer chooses an offer (x_{t+1}, s_t).

3. The respondent accepts or rejects the offer.

4. If accepted, then the proposal is implemented, and the game proceeds to the next period. If rejected, then player A obtains x_t and player B obtains $1 - x_t$. Upon rejection, Nature chooses:

    (a) With probability ϵ, cost κ is imposed on the proposer and the division of authority in the next period remains at x_t.

    (b) With probability $1 - \epsilon$, the proposed authority x_{t+1} is implemented.

Equilibrium

A state of the game is defined by the current division of authority x_t and the status quo policy s_{t-1}. Players choose among Markov strategies. This restricts players to condition strategies on the state of the game, and removes the potential for credible commitments across states.

The two possible "final" states of the game are $x = 0$ and $x = 1$, where player B has complete authority in the former, while A has complete authority in the latter. In the proceeding analysis, it will be shown that both $x = 0$ and $x = 1$ are absorbing states. However, the analysis investigates conditions for which A accumulates authority and the game ends up at $x = 1$. In accordance with this, the first-period status-quo policy is $s_0 = 0$, which capture B's initial advantage.

Summary

The exogenous parameters in the game are ρ, ϵ, δ, and κ. The choice variables are s_t and x_{t+1}, which the proposer in period t chooses, while the respondent chooses to accept or reject.

Comments on model assumptions

This paper draws upon a conception of authority as focal points for actors to coordinate around (Mailath et al. 2017; Myerson 2004). If authority is defined in this way, then many different relationships of authority are possible—whatever division of authority is coordinated upon. In the model, authority is conceived of as the right to unilaterally set policy up to some threshold (for a similar operationalization of the concept of authority, see Dragu and Polborn 2013). The scope of policymaking authority of an actor is determined by the division of authority across institutions.

Substantively, consider a constitutional government such as that of the United States. There is wide agreement that Congress and the President have distinct powers under the Constitution, which cannot be violated by the other branches. For instance, the President commands the military, but Congress has the authority to declare war. Regardless of the specific empirical application, if one believes in a separation of powers constitutional system, then one believes that authority can be split between different institutions. Actors within either institution can obtain partial policy benefits unilaterally, but an actor can only obtain the whole pie if the other assents (for a non-degenerate division of authority).[4]

Of course, the bounds of such authority are inevitably ambiguous and contested. Examples of constitutional ambiguities include Supreme Court judicial review, which was controversial for much of the 19th century, the Presidential veto as a policy and not just a constitutional tool (McCarty 2009, Latimer 2017), and the authority of colonial legislatures relative to imperial officials in 17th and 18th century America (Greene 2011). In the model, while both actors dividing the pie know what the formal boundary of authority is, there is some uncertainty, represented by $\epsilon < 1$, over whether the existing boundary of authority will be enforced. The probability that a claim to authority is successful despite the respondent's rejection could be very small. This might arise from a failure of collective action or simply inattentiveness on the part of the citizenry or other audience enforcing the division of authority between institutions. In the case in which a prior claim of authority is not publicly sanctioned, then it becomes the basis for future precedent.[5]

There are a few points to make about the use of random proposal power, which follows from games such as Baron and Ferejohn's (1989) classic model of legislative bargaining. The parameter ρ allows a biased rotation of proposal power between actors. On a theoretical level, for a single actor to have all proposal power is a strong assumption. While this may be plausible in some circumstances, there are many cases in which this is implausible. In

4. In contrast, non-constitutional institutions, such as bureaucratic agencies, can be created or destroyed by Congress. A standard account of delegation does not represent a transfer of authority in the sense used in this paper. Because any "policymaking authority" that Congress has granted to an agency is revocable and contingent on Congress's subsequent approval, Congress has not lost any authority in the sense in which I am using the term. Whereas Congress could abolish the Interstate Commerce Commission (ICC), Congress does not have the legal right to abolish constitutional offices such as President or Vice President. In this sense, then, Congress had complete authority over the ICC, but Congress has only limited authority relative to the President.

5. One might imagine that actors strategically make incremental claims to authority in order to diminish the likelihood that violations of precedent are noticed. This is empirically plausible, but is not necessary for a gradual transfer of authority to occur because the total size of the pie operates as a constraint on the move of the division of authority each period.

particular, we are substantively concerned with cases, such as legislatures in an authoritarian regime or international courts, in which a weak, subordinate institution accrues authority and eventually surpasses an initially more authoritative institution. Various actors that ultimately lost authority over time did in fact have opportunities to propose to increase their own authority, just as their bargaining partners did. It is something to be explained why one actor appears to be more aggressive in obtaining authority than another, and this is not possible (by this mechanism) if one actor never has the opportunity to offer a proposal.

Analysis

It is worth pausing to emphasize a fundamental point. Because the game is constant-sum and players have a common discount factor, should not the players be indifferent between all possible trades of policy-now versus policy-later? In fact, we will see that player A strictly prefers to accumulate authority. The reason for this arises from the role of noise, represented by ϵ, in the boundaries of authority between each actor. As long as there is some degree of noise associated with the enforcement of existing bounds of authority, then there are circumstances in which at least one player strictly prefers to sacrifice present policy in order to accumulate authority and obtain secure policy benefits in the future.

To understand why this is, consider the following. Suppose A is the proposer. Then B's condition to accept A's offer of (x_{t+1}, s_t) is

$$1 - s_t + \delta V_B(x_{t+1}) \geq 1 - x_t + \delta \left(\epsilon V_B(x_t) + (1-\epsilon) V_B(x_{t+1}) \right) \tag{1}$$

Here, V_i represents the continuation value for player i. The value x_t is the scope of authority for A in the current period. Upon rejection, A withdraws any policy benefits that they might have been willing to leave to B upon acceptance of the proposal. Importantly, there is ϵ

probability that A's proposed shift in authority "sticks" even if B rejects A's proposal. This creates an incentive for A to move authority as far as possible, since in doing so, A reduces the reservation value for B. This much utilizes the same mechanism as that of Powell's (2013) model.

However, consider the choice of a player to move authority or not. Here is the condition for player B to offer $s_t = 1$, consuming the entire pie today while keeping authority where it is (LHS), or to offer $s_t = 0$, forgoing policy benefits today and obtaining a more favorable distribution of future authority (RHS).

$$1 + \delta V_B(x_t) \geq 0 + \delta V_B(x_{t+1}) \tag{2}$$

The parameter ϵ does not appear in B's condition to offer $s = 1$ or $s = 0$ because, assuming that no player makes an offer that is sure to be rejected, on the RHS (where B gives us current policy benefits in order to move authority in B's favor) the offer of authority is such that A would accept, and the division of authority would therefore move for sure. We can see that in making an offer to A, player B faces a trade-off, since $\delta V_B(x_t) < \delta V_B(x_{t+1})$ while of course $1 > 0$. Whether or not this condition holds will depend on the relative magnitude of how B's continuation values change given B foregoing policy benefits today. The fact that there may be an asymmetry between A and B in this regard depends on the ρ (the probability of A being the proposer in a period) since $V_B(x_t) = \rho V_B^A(x_t) + (1 - \rho) V_B^B(x_t)$, with superscripts indicating which player is drawn to be proposer in the next period.

To illustrate the point, suppose that (magically) $\rho = 1$ at the penultimate period, so that player B (despite being the proposer in the current period) knows for sure that player A will be the proposer in the next period and will then offer a proposal that B will accept, moving the game to the final state and securing all future policy benefits. What should B do? Consider B's choice, in the case in which they can at most move authority in their own favor to state x_2 (where it takes at most two periods for A to accumulate remaining authority): Should B accept policy benefits today, then B can expect a payoff of $1 + \delta V_B^A(x_1)$, where $V_B^A(x_1) \leq 1$ since by definition of state x_1 player A can move the game to the final state from there by offering $\bar{s} \leq 1$. On the other hand, B can obtain $\delta(1 + \delta V_B^A(x_1))$ by choosing $s_t = 0$ and moving authority as far as possible away from state $x = 1$. But clearly the latter payoff is smaller than the former. The following analysis shows that this result also occurs under less restrictive assumptions.

Preliminaries

In this game, I analyze the case in which players never choose to initiate open conflict over boundaries of authority. By this, I mean that since the game is perfect information, actors always know whether an offer will be accepted or rejected. I assume that the cost of punishment in the case of an actor violating authority and their proposal being rejected is sufficiently high that they never do so.

Assumption 1. $\kappa > \bar{\kappa}$

Because punishment is incredibly costly, no actor will ever make an offer that they know will be rejected. In this way, actors are restricted to only make offers they know will be accepted.

In this analysis, we focus on a type of outcome of the game that I label a "developmental equilibrium", in which player A accumulates authority at the expense of player B. Initially though, it is convenient to establish a couple preliminary results. First, it is apparent that we may focus on players' preference for either the maximum or minimum value of s, since player utilities for policy and authority are linear. Consider each player's choice of x_t given that that player is the proposer and the respondent accepts.

If player A is the proposer, then B's condition to accept is

$$1 - s_t + \delta V_B(x_{t+1}) = 1 - x_t + \delta(\epsilon V_B(x_t) + (1-\epsilon)V_B(x_{t+1})) \tag{3}$$

$$1 - s_t = 1 - x_t + \delta\epsilon\left(V_B(x_t) - V_B(x_{t+1})\right) \tag{4}$$

So now we can think about the maximization problem for player A in terms of a single variable x_t, assuming that A prefers to induce acceptance by B.

$$s_t + \delta V_A(x_t) \tag{5}$$

The continuation values V_i are all linear in x_t in a single period, and all periods are added together. This implies that the second derivative with respect to x_t equals zero. Therefore, if a player prefers any value of (x_{t+1}, s_t), they always prefer a corner solution. This implies that (for the most part) we only need to check players' choices of $s_t = 0$ and $s_t = 1$ (if a player prefers either corner over the other, then that will be the optimum because no interior value will be preferable). The exception is the state immediately prior to a player moving the game to $x = 0$ or $x = 1$.

I refer to $x = 0$ and $x = 1$ as "final states" of the game because we can establish that if the game is in a state in which $x = 0$ or $x = 1$, then there are no further possible moves of authority. This is stated in the following lemma. (All proofs are in appendix.)

Lemma 1. *The states in which $x = 0$ and $x = 1$, with policy $s_{t-1} = 0$ and $s_{t-1} = 1$ respectively, are absorbing states.*

Once a player holds authority over the entire policy space, there is no offer to retract authority that that player accepts, since they can provide themselves current policy benefits of value

1. We use x_n to represent the division of authority from which it will take player A n periods to accumulate all authority (on the equilibrium path).

With these initial results, we can consider a possible strategy for player A. In such a hypothesized strategy, A will transfer all current policy benefits to B in exchange for a maximum possible shift in authority each period until the game reaches state $x = 1$. At this point, A will have accumulated all authority and obtain policy benefits of 1 each period, while B obtains zero forevermore.

A developmental equilibrium

In this section, we set out to find conditions for a developmental equilibrium. In such an equilibrium, one player gradually accumulates authority at the expense of the other player. The player losing authority allows this to happen (despite fully realizing what is happening) and consumes their share of today's pie rather than trading it for the possibility of future authority (and hence pie in the future).

Definition 1. *A* ***developmental equilibrium*** *is an equilibrium with the following properties:*

- *At every period in which A is the proposer, except for the penultimate period x_1, player A chooses $s_t = 0$.*
- *When A is the proposer at the penultimate period x_1, A offers $\bar{s} \in (0, 1]$ to B so that B accepts, moving the game to $x = 1$.*
- *At every period in which B is the proposer, player B chooses $s_t = 0$.*
- *After incrementally moving toward $x = 1$ (whenever A is the proposer), the game remains stable forever at $x = 1$, with player A receiving 1 every period and player B receiving 0.*

We first characterize the equilibrium path, then show that neither A nor B have an incentive to deviate. Finally, we will see that for $\delta > \bar{\bar{\delta}}$, $\epsilon > \bar{\epsilon}$, and $\rho > \bar{\rho}$, where $\bar{\bar{\delta}}$, $\bar{\epsilon}$, and $\bar{\rho}$ are defined in the proof of Lemma 3, this equilibrium path occurs in all possible equilibria.

Lemma 2. *Assuming that B takes current policy benefits in every state prior to $x = 1$, player A chooses $s_t = 0$ in each state until x_1, at which point A chooses $\bar{s}$. When $x = 1$, player A chooses $s_t = 1$ every period.*

- *Player A's expected utility in state x_n on the equilibrium path is:* $\frac{\delta^{n-1}(x_1(1-\delta)+\delta(1-\epsilon))\rho^n}{(1-\delta)(1-\delta(1-\rho))^{n-1}(1-\delta-\delta(1-\epsilon)\rho)}$
- *On the equilibrium path,* $x_n = \frac{\delta^{n-1}(x_1(1-\delta)+\delta(1-\epsilon))\epsilon\rho^{n-1}}{(1-\delta(1-\rho))^{n-1}(1-\delta-\delta(1-\epsilon)\rho)}$

In Appendix B, I show that the sum of player A's expected utility in state x_n on the equilibrium path and player B's expected utility in state x_n, derived independently, sum to $\frac{1}{1-\delta}$. But we already know this because utilities are constant sum, so the portion of the pie that each player receives must be one minus the portion of the other player, and the total pie is $\frac{1}{1-\delta}$.

Finally, we have a somewhat technical but useful result, during the course of which we can define $\bar{\rho}$, $\bar{\epsilon}$, and $\bar{\bar{\delta}}$. Each of these are defined so that we can specify a parameter region in which the developmental equilibrium path occurs.

Lemma 3. *The following condition holds for $\rho > \bar{\rho}$, $\epsilon > \bar{\epsilon}$, and $\delta > \bar{\bar{\delta}}$.*

(6)
$$\frac{\delta^{n-1}\rho^{n-1}}{(1-\delta(1-\rho))^{n-1}} > \frac{\left(\frac{1}{\epsilon}-1\right)(1-\delta+\delta(1-\epsilon)\rho)}{(x_1(1-\delta)+\delta(1-\epsilon))}$$

A's strategy

Recall that we have assumed the status quo policy is $s_0 = 0$, meaning that policy starts out in B's favor. This means that from A's perspective, A can either use A's authority to move policy to x_t, which is the best policy (for A) that A can achieve given the limitation on A's authority, or A can maintain the status quo policy and offer to move authority in A's favor instead. By maintaining the status quo policy, A is using A's authority to set policy in B's favor. In effect, since A holds some amount of authority, A can threaten B to move policy away from what B favors, unless B grants A further policymaking authority.

Moreover, as the following result states, A strictly prefers to move authority rather than policy when $\epsilon < 1$. This is because moving authority, but not policy, could potentially be successful even if B rejects the offer. In contrast, if $\epsilon = 1$, then A is indifferent between moving authority and moving policy.

Proposition 1. *Assuming that B chooses $s_t = 0$ in every state other than $x = 1$, player A strictly prefers to move authority when A is proposer.*

B's strategy

The fact that a player is *not* obtaining current policy benefits creates an incentive to move authority when the opportunity arises, as we have shown with A's strategy. In contrast, when a player *is* obtaining current policy benefits, as B is on the equilibrium path, then that player faces a trade-off when the opportunity arises to move authority. By proposing to move authority, player B would be giving up current policy benefits. More than this, because B can preserve the status quo without obtaining A's assent, B is getting a better policy than B would be getting by setting a policy that A would accept.

We can see in the following result that B is willing to accept the trade-off of current policy over future authority as long as ϵ is sufficiently large. When ϵ is large, then ambiguity on authority is low. This means that B's opportunity to retract authority is relatively limited.

Proposition 2. *Assuming that A chooses $s_t = 0$ in every state prior to x_1, $\bar{s}$ in x_1, and $s_t = 1$ when $x = 1$, player B chooses $s_t = 0$ in every state, as long as $\epsilon > \bar{\epsilon}$*

It is notable that the condition on ϵ for the developmental equilibrium path to occur comes from player B's strategy, since any value of epsilon is sufficient for A to strictly prefer to move authority.

Historically, it is plausible to imagine ϵ being indexed for each player. A population enforcing the bounds of authority through protests or otherwise may be more sensitive to violations by some actors relative to others. This is the claim of some historians regarding the case of colonial American assemblies, that colonists were much more sensitive to usurpations of authority by imperial officials than by colonial legislators (Bliss 1990, Greene 2011). Incorporating this into the model would increase the potential for player A to increase authority. But I have left ϵ common across players to emphasize that two players are in very similar starting positions, with the only asymmetries being the status quo policy and the degree of proposal power.

The trajectory of authority

Having established A and B's strategies, contingent on what the other player is doing, we will now combine these to investigate possible equilibrium strategy profiles. The following proposition tells us that in the parameter region of interest, the developmental equilibrium path is unique.

Proposition 3. *For any equilibrium strategy profile, there is an equilibrium path in which player A accumulates all authority as long as $\rho > \bar{\rho}$, $\epsilon > \bar{\epsilon}$, and $\delta > \bar{\bar{\delta}}$.*

Two elements of the model are at the foundation of this result. The first is that A has a dominant strategy to move authority for any value of ϵ and status quo policy less than A's limit on authority. The second element that matters here is that the initial status quo policy is in B's favor. We justified this assumption in the model setup based on B being the prevailing authoritative institution under the initial conditions in which the game begins. Yet now we see that it is exactly this supposedly favorable condition for B which creates an incentive both for B to not retract authority and for A to accumulate authority.

Figure 1 illustrates possible trajectories of authority for different parameter values (all of which fulfill the criteria that $\rho > \bar{\rho}$, $\epsilon > \bar{\epsilon}$, and $\delta > \bar{\delta}$). As can be seen, the parameter that matters the most for the speed at which authority is accumulated is δ. Straightforwardly, δ controls the value of current policy relative to future policy. Hence when δ is high player B requires relatively more present policy to make up for future policy loss, and therefore the overall speed at which A accumulates authority is slower when δ is relatively large.

Perhaps surprisingly, different values of ρ do not make a difference for the trajectory of authority, at least in expectation. Within the constraint that $\rho > \bar{\rho}$, player A compensates for the probability that B will be the proposer while authority is being accumulated with the size of the increments that A offers to B. In sum, this achieves roughly the same dynamic trade of policy now for policy later across values of ρ. Because, unlike δ, ρ does not change the value of policy now relative to policy later, the overall quantity that A needs to provide to B in order for B to accept A's accumulation of authority remains the same.

But the constraint that $\rho > \bar{\rho}$ is nonetheless important. We can interpret ρ to represent that over the course of a span of time, one institution will be more effective at generating proposals than another, perhaps due to internal veto points or collective action problems (that is, substantive institutional features) on the part of either institution.

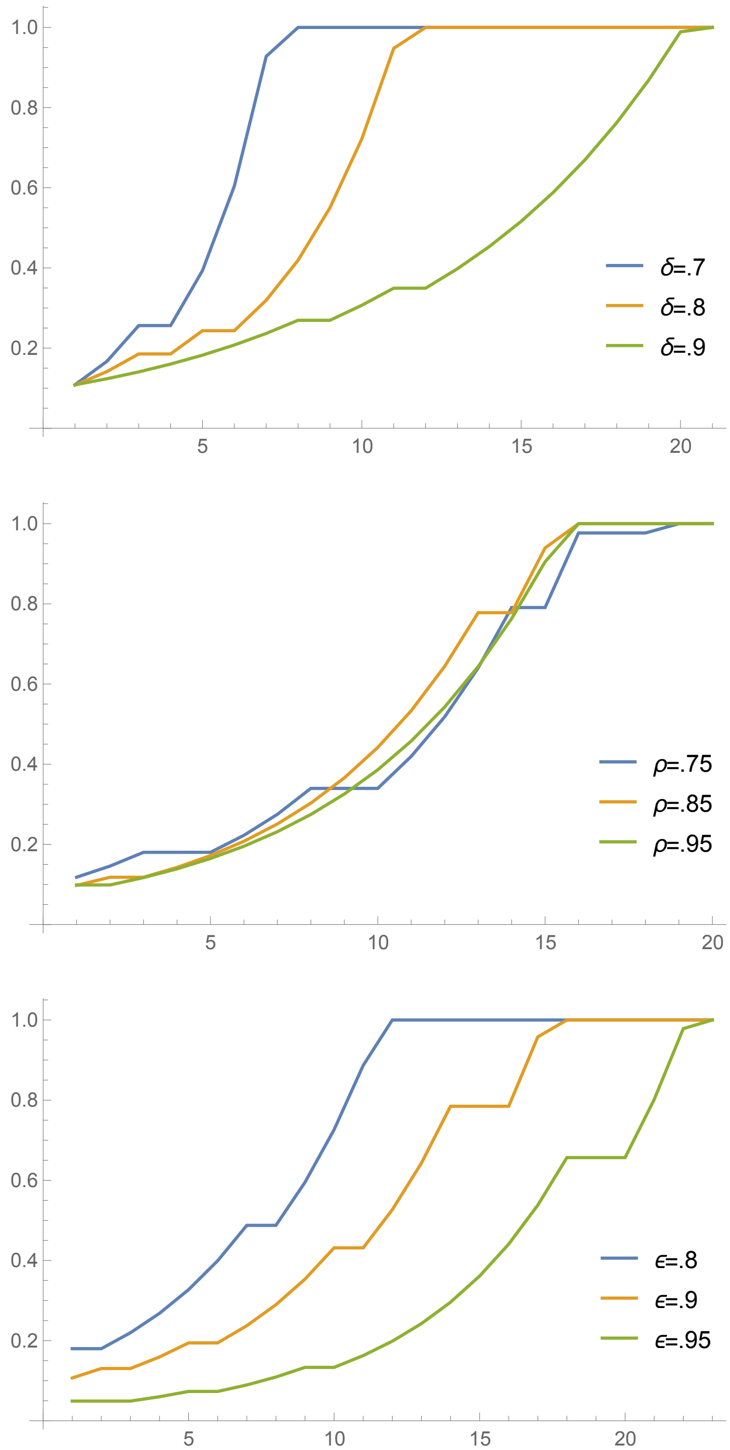

Figure 1: Each graph starts at the lowest possible value for the division of authority, that is, the most favorable division for player B. So in subfigure c, the different lengths of the trajectories are somewhat deceptive, in that they take about the same number of period from the same point. Parameter values are $\delta = 0.85$, $\epsilon = 0.9$, $\rho = 0.8$, unless otherwise stated.

Historical case study

At the onset of the American Revolution in 1775, the territory of British North America and the Caribbean was divided into 26 political units, each with a legislative assembly. The history of colonial assemblies is one of continual conflicts between colonists acting through assemblies and the crown or their representatives. The assemblies maintained powers to tax and allocate colonial budgets, and this inhibited crown revenue, trade policy, and spending on imperial defense. On issues of taxation and spending for imperial defense, assemblies gave voice and political power to colonial interests opposed to crown objectives. Assemblies leveraged their control over revenue to expand power in other domains, such as appointment of judges (Greene 1963).

Though colonists were reluctant to explicitly ignore crown rules, they frequently exploited legal technicalities or contested what was legal (Stanwood 2011). Metropolitan officials were vulnerable to colonial refusals to comply with their demands. The cost that colonial protests could impose might come directly through tax resistance, through the disruption of economic activity resulting from a ceasing of normal operations of government, or (ultimately) the expense of an imperial military expedition. Colonial protests were especially costly for the crown during wartime, when colonial non-cooperation could mean military defeat.

The capacity to develop policy

Throughout the colonial period, imperial officials struggled to develop a coherent colonial policy. In the 17th century, this was primarily due to financial stress, political turbulence, and civil war in England. For much of the 18th century, this pertained to political divisions between the King's Privy Council and Parliament, as well as the colonies appearing to be a relatively low priority for Parliament. Overall, the low capacity of imperial officials to develop policy corresponds to a high value of ρ in the model, since the result is that colonial legislatures had a greater number of opportunities to propose policy to imperial officials.

For much of the 17th century, the English state lacked resources to formulate a coherent colonial policy. One historian writes, “Although a Commission for Regulating Plantations was set up under the chairmanship of Archbishop Laud in 1634, the crown was not strong enough, and the colonial economies themselves not developed enough, to allow the imposition of any significant degree of uniformity, or even of central direction.” Only with increased state capacity under the Commonwealth and after could imperial officials, “think in practical terms of developing a genuinely imperial policy and a more systematic framework for the government of overseas empire,” (Elliott 2006, 118). Elliott contrasts Archbishop Laud’s commission with the Spanish “Council of the Indies. . . a central organ for the formulation and implementation of policy relating to every aspect of the life of its American possessions,” and suggests that only with the establishment of the Board of Trade in 1696 did England gain a “remotely equivalent” agency (2006, 122-3).

After the Glorious Revolution in 1688, the Board of Trade sought to exert greater direct control over the colonies through the resumption of proprietary charters. Due to post-1688 political realities, this was to be accomplished through a general resumption bill in Parliament. The Board’s failed to achieve its legislative goals, seemingly due to the low priority of the colonies for Parliament. Notwithstanding vigorous support for resumption by the Board of Trade in the early 18th century, the resumption bill “slipped off the busy order paper of the House of Lords despite a vote that revealed the intention to proceed further with the matter,” (Steele 1986, 246). Steele explains the failure of the Resumption Bill by its low priority for Parliament (Steele 1968, 79). The Board continued to seek Parliamentary action on resumption, with specific efforts in 1702, 1706, 1715, and 1722, without success (Steele 1968, 81).

Despite setbacks in Parliament, the Board of Trade persevered in their efforts to more closely manage the colonies. The Board continued to pursue royalization when opportunities arose in specific colonies. In 1706, the Board of Trade issued instructions to all royal governors to send drafts for approval or include a suspending clause in "all laws that affected the royal prerogative or the private property of subjects," (Steele 1986, 233). This built on an earlier Stuart policy in Pennsylvania and a similar policy in Massachusetts. The Board included further categories of legislation in following years (Steele 1986, 234). Having failed to abolish assemblies or to remove their agenda-setting power through Poynings method, the main strategy left to imperial officials was to veto assembly legislation. In this way, they sought to protect their colonial interests.

There are three occasions (in 1734, 1744, and 1749) when, as Greene (1963, 17) writes, "the ministry failed to give enthusiastic support to measures introduced into Parliament to insure the supremacy of instructions over colonial laws." One interpretation of this is that officials on the Privy Council were opposed to Parliamentary efforts to limit assembly power (Gailmard 2017, 8). Another interpretation is that these colonial measures were a low priority for Parliament and duplicative of policies that the Board of Trade was already pursuing. Knollenberg suggests that protests by colonial agents in London could have been a factor for failure of Parliamentary action on the proposed legislation in 1734 (1960, 49). As for 1744 and 1749, Parliament adjourned without action despite indications that the measures would have been approved, suggesting that these were not a high priority for Parliament. This provides an example of how agency problems, internal politics, or a finite agenda inhibited the ability of metropolitan officials to restrain efforts by assemblies to increase authority.

Ambiguity of authority

The other important parameter of the model is ϵ, representing ambiguity in the division of authority between institutions. Greene (2011) emphasizes the difference in perspectives between American colonists and imperial officials concerning colonial legislative authority. Part of this goes back to English traditions of government power, as articulated in the

17th century works of James Harrington and Edward Coke. In this view, authority was split across multiple institutions, including the King, Parliament, and common law courts. Colonists drew on this tradition to argue that colonial legislatures held authority independent of the crown or Parliament.

As Bliss (1990) points out, colonists even designed such apparently peripheral elements of government, such as the architecture of the assembly buildings, in order to convey the authority of the assemblies. Figure 2 shows an example of this, where the design of the Virginia House of Burgesses chamber emulates that of St. Stephen's Chapel, where the House of Commons met at the time. Such features such as these presumably contributed to ambiguities over the divisions of authority between colonial and metropolitan institutions.

Figure 2: Reconstructions of the interiors of the House of Commons chamber (left) and the Virginia House of Burgesses (right).

To provide an example of how these ambiguities over authority affected imperial policy, consider a 1670s attempt by the crown to impose Poynings Law in Jamaica (Elliott 2006, 150; Steele 1986, 232). Also called the "Irish method", this would have restricted assembly meetings to prior royal approval, the crown would have prior approval on assembly bills, and the crown could amend bills, on which the assembly could then only vote yes or no.

Jamaica's resistance to the crown illustrates the difficulties facing the crown in achieving its ends. Thomas Lynch, previous (and future) lieutenant-governor of Jamaica, writes to the Lords of Trade regarding the Jamaican Assembly's response to the crown's demands, "This they have found grievous and inconvenient, and have addressed Lord Carlisle to intercede with His Majesty to change these orders, which, as I hear, His Majesty, on report of the Committee, has not consented to do."[6] Lynch describes the numerous reasons that the colonists cite in defense of their position, including "That being English they think they have a right to be governed as such, and to have their liberties and properties secured by the laws of England, or others of their own making."

After listing colonial justifications for their resistance, Lynch provides his perspective. It is worth quoting at length to show the factors considered relevant by imperial officials in fighting or accepting assembly authority.

> The Assembly will probably reject the laws offered to them, yet the need for revenue is urgent; the Council may join the Governor to order the laws to be continued, but I verily believe that they will not continue the Revenue Bill, for they think that belongs to the Assembly. If they do it, it will not be without process, and I doubt the Judges would quit and the juries give constantly against the officers. It would be the same, or worse, if an order to that effect were sent from England, and it would give strange umbrage to the rest of the colonies, which are too much discouraged already by low prices and French competition.

The use of "doubt" here is likely to be in an older sense of the word to mean "am uncertain about whether" rather than the modern connotation of "deem it unlikely that". One can observe Lynch's fears of colonial protests both among elites (judges) and citizens (juries). Lynch's concerns are reasonable; widespread civil resistance by juries and legal officials in response to perceived metropolitan usurpations occurred, such as in New York in the 1760s

6. Sir Thomas Lynch to Lords of Trade and Plantations. Sir Thomas Lynch. The Calendar of State Papers, Colonial: North America and the West Indies 1574-1739, 10.(Dec 18, 1679). https://search.proquest.com/csp/docview/1845174723/F6D99174BA35405APQ/8?accountid=14496

(Reid 1977). The Jamaican Assembly ultimately approved several 20 year revenue bills and then a permanent revenue in 1728, though Poynings Law was never imposed.

Implications for understanding institutional development

At an abstract level, institutions are conceived of as durable sets of rules that shape actor behavior (North 1990, Sveinmo and Thelen 1992, Pierson 2004). But given that institutions—from which strategic incentives or norms of appropriate behavior arise—are seen as the source of explanation for political outcomes, how is it that institutions are transformed over time? This is the basic problem confronting scholars of institutions.

An older approach rested explanatory power upon "critical junctures", which were posited to be historical periods in which institutional arrangements were up for grabs, and alternative future paths were possible. While most historical and political events are shaped by institutions, the argument went, during critical junctures, institutions are shaped by historical and political events. This view has been criticized, such as by Sveinmo and Thelen (1992), who memorably characterized the view as "institutions explain everything until they explain nothing."

Other efforts to explain institutional transformation rest, essentially, upon some portion of actors ignoring some elements of the game. In Bednar and Page (2018), the strategic environment changes, but only a portion of actors update their strategies. In Greif and Laitin (2004), actions within the game have effects that are not taken into account by the actors playing the game. Greif and Laitin refer to these effects as "quasi-parameters". While it may be the case that both of these perspectives capture important elements of the world, they leave for further investigation questions of exactly what elements of institutions result in changes to the strategic environment or the nature of quasi-parameters (Thelen 2005).

For this reason, scholars such as Greif and Laitin (2004) and Streeck and Thelen (2005) have argued for increased attention to endogenous institutional change in which ongoing "everyday" interactions among political actors alter the institutions of which they are a part. Ideally, theories of endogenous institutional change would explain how actors can strategically yet gradually transform institutional relationships, and shed light upon the conditions under which such an outcome is possible. In game-theoretic terms, this entails the movement of key institutional features being choices by actors in the model. Yet models in which endogenous institutional change occurs gradually tend to involve some violation of strategic decision-making by players in the game (e.g., Greif and Laitin 2004, Bednar and Page 2018).

The model presented herein shows how such ambiguity over authority, in a context of policy bargaining, can create conditions for actors to trade policy for authority. In a constitutional system in which authority is split across institutions, it is likely inevitable that there are ambiguities over the precise division of authority. Stephen Skowronek poetically describes "cracks in an edifice of rules of action" (1982, 287). The overarching suggestion from Skowronek, Pierson (2004), and others is that ambiguity can be a motor of institutional development. My contribution is to take this idea to the level of strategic incentives.

Conclusion

This paper has argued that ambiguities in the division of authority across institutions can create incentives for one institution to accrue authority at the expense of the other. This can result in an institution that initially had nearly zero authority eventually establishing complete authority over the other institution. This change occurs gradually as the two institutions engage in a dynamic bargain of current policy for future authority. The role of ambiguity is crucial because it allows one institution to decrease the reservation value of the other when offering a trade of policy for authority, and thereby creates an incentive for the proposing institution to grant policy now in exchange for authority later.

The model presented in this paper analyzed this strategic situation. In the model, every period two actors (representing institutions) have an opportunity to change the division of authority and set policy. The analysis of the model showed conditions under which the authority of one actor gradually increases at the expense of the other. For the institution for whom authority is being dissipated, two factors make that institution unwilling or unable to respond. First, that institution benefits from the status quo policy, which creates an incentive to maintain it. Second, there is a small enough degree of ambiguity that the other institution is unwilling to accept a retraction. For this mechanism to occur, there must be a sufficient difference in the proposal power of the two institutions (interpreted as the capacity to develop policy), but proposal power is nonetheless shared between both institutions.

Finally, I have used the historical case of the development of colonial assemblies in British America to illustrate the model. It has been widely observed that assemblies established authority over an increasing number of policy domains during the colonial period. The model provides an explanation for this observation. In the historical discussion, I provided evidence that imperial officials struggled to formulate a coherent colonial policy, in part due to metropolitan state capacity and in part due to divisions within the British political system. I also provided evidence of ambiguity over the division of authority between colonial assemblies and imperial officials. Together, these factors are theoretically sufficient to explain how colonial assemblies were able to accumulate authority at the expense of imperial officials. This is a different approach to theorizing electoral institutions in authoritarian regimes, where the usual practice is to ask how the power of electoral institutions solve an optimization problem on the part of the ruler (Gehlbach et al 2016). An important question for future research concerns when authority is truly divided within a regime, or alternatively when electoral institutions persist solely at the sufferance of the ruler.

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Supporting information for “Stealing authority: A model of institutional transformation”

November 10, 2020

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A Formal proofs

Proof of Lemma 1: Let a represent player i's expected utility from state g as a portion of $\frac{1}{1-\delta}$, and let b represent player i's expected utility from state h as a portion of $\frac{1}{1-\delta}$, with $a < b$. In order for authority to be moved from state h to state g, player i must accept j's proposal. In order for player i to accept j's proposal, the following condition must hold:

$$1 + \frac{\delta}{1-\delta} a \geq 1 + \frac{\delta}{1-\delta}(\epsilon b + (1-\epsilon)a) \tag{7}$$

This condition never holds for any $0 < a < b < 1$, $0 < \delta < 1$, or $0 < \epsilon < 1$. Hence, when a player has authority over the entire policy space, they never accept an offer to increase the other player's authority. Therefore, states in which $x = 0$ or $x = 1$ are absorbing states. $\square$

Proof of Lemma 2: For each case, we will show values of x_n and A's expected utility.

Case 1: B's condition to accept at x_1 is

(8)

$$\bar{s} + \delta(\rho 0 + (1-\rho)0) = 1 - x_1 + \delta\left(\epsilon\left(\rho\bar{s} + (1-\rho)\frac{1+\bar{s}\delta\rho}{1-\delta(1-\rho)}\right) + (1-\epsilon)(\rho 0 + (1-\rho)0)\right)$$

This condition implies that $\bar{s} = 1 - \frac{x_1(1-\delta(1-\rho))-\delta\epsilon}{1-\delta(1-\rho(1-\epsilon))}$. Because $\bar{s}$ must be in $[0,1]$, this necessitates that $x_1 \geq \frac{\delta\epsilon}{1-\delta(1-\rho)}$. For there to be any possible x_1, then, it must be the case that $\frac{\delta\epsilon}{1-\delta(1-\rho)} < 1$. This last condition holds as long as $\rho > \frac{\delta+\delta\epsilon-1}{\delta}$, which equals $\bar{\rho}$ as defined in Lemma 3.

Player A's expected utility from B accepting (when A is the proposer) is

$$1 - \bar{s} + \frac{\delta}{1-\delta} \tag{9}$$

$$1 - \left(1 - \frac{x_1(1-\delta(1-\rho)) - \delta\epsilon}{1-\delta(1-\rho(1-\epsilon))}\right) + \frac{\delta}{1-\delta} \tag{10}$$

$$\frac{\delta}{1-\delta} - \frac{x_1(1-\delta(1-\rho)) - \delta\epsilon}{1-\delta+\delta(1-\epsilon)\rho} \tag{11}$$

Prior to the proposer being drawn, A's expected utility in state x_1 is

$$V_A(x_1) = \rho\left(\frac{\delta}{1-\delta} - \frac{x_1(1-\delta(1-\rho)) - \delta\epsilon}{1-\delta+\delta(1-\epsilon)\rho}\right) + (1-\rho)\delta V_A(x_1) \tag{12}$$

$$= \frac{(x_1+\delta-x_1\delta-\delta\epsilon)\rho}{(1-\delta)(1-\delta+\delta(1-\epsilon)\rho)} \tag{13}$$

Case 2: B's condition to accept at x_2 is

$$1 + \delta\left(\rho\bar{s} + (1-\rho)\frac{1+\bar{s}\delta\rho}{1-\delta(1-\rho)}\right) = 1 - x_2 \tag{14}$$
$$+\delta\left(\epsilon\left(\rho\frac{1+\bar{s}\delta\rho}{1-\delta(1-\rho)} + (1-\rho)\frac{1+\delta(-1+\rho(2+\bar{s}\delta\rho))}{(1-\delta(1-\rho))^2}\right) + (1-\epsilon)\left(\rho\bar{s} + (1-\rho)\frac{1+\bar{s}\delta\rho}{1-\delta(1-\rho)}\right)\right)$$

For this condition to hold,

$$x_2 = \frac{(1-\bar{s}(1=\delta))\delta\epsilon\rho}{(1-\delta(1-\rho))^2} \tag{15}$$

$$= \frac{\delta(x_1(1-\delta)+\delta(1-\epsilon))\epsilon\rho}{(1-\delta(1-\rho))(1-\delta+\delta(1-\epsilon)\rho)} \tag{16}$$

Once A has been drawn as the proposer, A's expected utility is $\delta\left(\frac{(x_1+\delta-x_1\delta-\delta\epsilon)\rho}{(1-\delta)(1-\delta+\delta(1-\epsilon)\rho)}\right)$.

Prior to the proposer being drawn, A's expected utility in state x_2 is

$$V_A(x_2) = \rho\delta\left(\frac{(x_1+\delta-x_1\delta-\delta\epsilon)\rho}{(1-\delta)(1-\delta+\delta(1-\epsilon)\rho)}\right)+(1-\rho)\delta V_A(x_2) \tag{17}$$

$$= \frac{(x_1+\delta-x_1\delta-\delta\epsilon)\rho^2}{(1-\delta)(1-\delta(1-\rho))(1-\delta+\delta(1-\epsilon)\rho)} \tag{18}$$

Now, we will use the values of x_2 and A's expected utility at x_2 as base cases for inductive arguments about x_n and A's expected utility at every state.

The value of x_n in every state. We can use a proof by mathematical induction to show that $x_n = \frac{\delta^{n-1}(x_1(1-\delta)+\delta(1-\epsilon))\epsilon\rho^{n-1}}{(1-\delta(1-\rho))^{n-1}(1-\delta-\delta(1-\epsilon)\rho)}$. Assume that $x_n = \frac{\delta^{n-1}(x_1(1-\delta)+\delta(1-\epsilon))\epsilon\rho^{n-1}}{(1-\delta(1-\rho))^{n-1}(1-\delta-\delta(1-\epsilon)\rho)}$ to show $x_{n+1} = \frac{\delta^{n}(x_1(1-\delta)+\delta(1-\epsilon))\epsilon\rho^{n}}{(1-\delta(1-\rho))^{n}(1-\delta-\delta(1-\epsilon)\rho)}$. Again, the relevant condition is B's condition to accept A's offer. We write out this condition in terms of $V_B^A(x_{n+1})$ and x_n.

$$\begin{aligned} 1+\delta\left(\rho V_B^A(x_{n+1})+(1-\rho)\frac{1+V_B^A(x_{n+1})\delta\rho}{1-\delta(1-\rho)}\right) \\ = 1-x_n+\delta\epsilon\left(\rho\frac{1+V_B^A(x_{n+1})\delta\rho}{1-\delta(1-\rho)}+(1-\rho)\frac{1+\delta(-1+\rho(2+V_B^A(x_{n+1})\delta\rho))}{(1-\delta(1-\rho))^2}\right) \\ +\delta(1-\epsilon)\left(\rho V_B^A(x_{n+1})+(1-\rho)\frac{1+V_B^A(x_{n+1})\delta\rho}{1-\delta(1-\rho)}\right) \end{aligned} \tag{19}$$

This condition is true when $x_n = \frac{(1-V_B^A(x_{n+1})(1-\delta))\delta\epsilon\rho}{(1-\delta(1-\rho))^2}$. We can rewrite this as $V_B^A(x_{n+1}) = \frac{x_n(1-\delta(1-\rho))^2-\delta\epsilon\rho}{-(1-\delta)\delta\epsilon\rho}$.

We also find an expression for x_{n+1}, which is the state that is one increment farther from x_0 than x_n, also as a function of $V_B^A(x_{n+1})$.

$$
\begin{aligned}
(20)\quad & 1+\delta\left(\rho\frac{1+V_B^A(x_{n+1})\delta\rho}{1-\delta(1-\rho)}+(1-\rho)\frac{1+\delta(-1+\rho(2+V_B^A(x_{n+1})\delta\rho))}{(1-\delta(1-\rho))^2}\right)=1-x_n \\
&+\delta\epsilon\left(\rho\frac{1+\delta(-1+\rho(2+V_B^A(x_{n+1})\delta\rho))}{(1-\delta(1-\rho))^2}+(1-\rho)\frac{1+\delta(-2+\delta+3\rho+\delta\rho(-3+\rho(3+V_B^A(x_{n+1})\delta\rho)))}{(1-\delta(1-\rho))^3}\right) \\
&+\delta(1-\epsilon)\left(\rho\frac{1+V_B^A(x_{n+1})\delta\rho}{1-\delta(1-\rho)}+(1-\rho)\frac{1+\delta(-1+\rho(2+V_B^A(x_{n+1})\delta\rho))}{(1-\delta(1-\rho))^2}\right)
\end{aligned}
$$

This condition is true when $x_{n+1}=\frac{(1-V_B^A(x_{n+1})(1-\delta))\delta^2\epsilon\rho^2}{(1-\delta(1-\rho))^3}$. We substitute the expression for $V_B^A(x_{n+1})$ into the expression for x_{n+1}.

$$
\frac{(1-\frac{x_n(1-\delta(1-\rho))^2-\delta\epsilon\rho}{-(1-\delta)\delta\epsilon\rho}(1-\delta))\delta^2\epsilon\rho^2}{(1-\delta(1-\rho))^3}=\frac{x_n\delta\rho}{1-\delta(1-\rho)} \tag{21}
$$

Having assumed that $x_n=\frac{\delta^{n-1}(x_1(1-\delta)+\delta(1-\epsilon))\epsilon\rho^{n-1}}{(1-\delta(1-\rho))^{n-1}(1-\delta-\delta(1-\epsilon)\rho)}$, and having shown that $x_{n+1}=\frac{x_n\delta\rho}{1-\delta(1-\rho)}$, we can see that

$$
x_{n+1}=\frac{\delta^{n-1}(x_1(1-\delta)+\delta(1-\epsilon))\epsilon\rho^{n-1}}{(1-\delta(1-\rho))^{n-1}(1-\delta-\delta(1-\epsilon)\rho)}\left(\frac{\delta\rho}{1-\delta(1-\rho)}\right) \tag{22}
$$

$$
=\frac{\delta^{n}(x_1(1-\delta)+\delta(1-\epsilon))\epsilon\rho^{n}}{(1-\delta(1-\rho))^{n}(1-\delta-\delta(1-\epsilon)\rho)} \tag{23}
$$

This is what we set out to show.

A's expected utility in every state. We need an induction proof to show A's expected utility for all states on the equilibrium path. We hypothesize that $V_A(x_n)=\frac{\delta^{n-1}(x_1(1-\delta)+\delta(1-\epsilon))\rho^n}{(1-\delta)(1-\delta(1-\rho))^{n-1}(1-\delta-\delta(1-\epsilon)\rho)}$ to show that $V_A(x_{n+1})=\frac{\delta^{n}(x_1(1-\delta)+\delta(1-\epsilon))\rho^{n+1}}{(1-\delta)(1-\delta(1-\rho))^{n}(1-\delta-\delta(1-\epsilon)\rho)}$. Prior to the proposer being drawn in x_{n+1}, A's expected utility is

$$V_A(x_{n+1} = \rho\delta V_A(x_n) + (1-\rho)\delta V_A(x_{n+1} \tag{24}$$

$$= \frac{\rho\delta V_A(x_n)}{1-\delta(1-\rho)} \tag{25}$$

$$= \frac{\delta^{n-1}(x_1(1-\delta)+\delta(1-\epsilon))\rho^n}{(1-\delta)(1-\delta(1-\rho))^{n-1}(1-\delta-\delta(1-\epsilon)\rho)} \frac{\rho\delta}{1-\delta(1-\rho)} \tag{26}$$

$$= \frac{\delta^{n}(x_1(1-\delta)+\delta(1-\epsilon))\rho^{n+1}}{(1-\delta)(1-\delta(1-\rho))^{n}(1-\delta-\delta(1-\epsilon)\rho)} \tag{27}$$

And this is what we set out to show. Therefore, we have shown the expected utility for every state on the equilibrium path. □

Proof of Lemma 3: Define $\bar{\rho} := \frac{\delta+\delta\epsilon-1}{\delta}$. Because ρ and ϵ are probabilities and must be in $[0,1]$, while the discount factor is defined as in (0,1), we also need $\frac{1}{1+\epsilon} < \delta < 1$ to guarantee that $\frac{\delta+\delta\epsilon-1}{\delta} > 0$. Hence, define $\bar{\delta} := \frac{1}{1+\epsilon}$.

Now we shall define $\bar{\epsilon}$ to be a value such that for any $\epsilon > \bar{\epsilon}$, the following condition holds

$$\frac{\delta^{n-1}\rho^{n-1}}{(1-\delta(1-\rho))^{n-1}} > \frac{\left(\frac{1}{\epsilon}-1\right)(1-\delta+\delta(1-\epsilon)\rho)}{(x_1(1-\delta)+\delta(1-\epsilon))} \tag{28}$$

The expression $\frac{\left(\frac{1}{\epsilon}-1\right)(1-\delta+\delta(1-\epsilon)\rho)}{(x_1(1-\delta)+\delta(1-\epsilon))}$ is decreasing in ϵ when $0 < \epsilon < 1$. Moreover, when $\epsilon = 1$, the expression equals zero. Because $\frac{\delta^{n-1}\rho^{n-1}}{(1-\delta(1-\rho))^{n-1}}$ is constant in ϵ, it is always possible to increase ϵ sufficiently for the condition to hold, no matter the values of δ and ρ.

Letting $\Gamma := \frac{\delta^{n-1}\rho^{n-1}}{(1-\delta(1-\rho))^{n-1}}$, we can solve $\Gamma = \frac{\left(\frac{1}{\epsilon}-1\right)(1-\delta+\delta(1-\epsilon)\rho)}{(x_1(1-\delta)+\delta(1-\epsilon))}$ for ϵ.

(29)

$$\bar{\epsilon} := \frac{1+\Gamma x_1 - \delta + \Gamma\delta - \Gamma x_1\delta + 2\delta\rho - \sqrt{4\delta(1-\delta(1-\rho))(\Gamma+\rho) + (-1-\Gamma x_1+\delta-\Gamma(1-x_1)\delta - 2\delta\rho)^2}}{2\delta(\Gamma+\rho)}$$

For $\epsilon > \bar{\epsilon}$, the above condition holds. $\square$

Proof of Proposition 1: We prove by induction that A strictly prefers to move authority on the equilibrium path. We start with x_1 as our base case. A's expected utility from offering $s = 1 - \frac{x_1(1-\delta(1-\rho))-\delta\epsilon}{1-\delta(1-\rho(1-\epsilon))}$ in state x_1 is

$$1 - \left(1 - \frac{x_1(1-\delta(1-\rho)) - \delta\epsilon}{1-\delta(1-\rho(1-\epsilon))}\right) + \frac{\delta}{1-\delta} \tag{30}$$

And A's expected utility from deviating to take current policy benefits for one period, then returning to the equilibrium path in the next period is

$$x_1 + \frac{\delta\rho\left(1 - \left(1 - \frac{x_1(1-\delta(1-\rho))-\delta\epsilon}{1-\delta(1-\rho(1-\epsilon))}\right) + \frac{\delta}{1-\delta}\right)}{1-\delta(1-\rho)} \tag{31}$$

When $\epsilon < 1$, the top expression is strictly greater than the bottom expression. (When $\epsilon = 1$, they are equal.)

Now for the inductive step of the argument. We assume that A's expected utility when A is the proposer in state x_n, denoted $V_A^A(x_n)$, is strictly greater than $x_n + \frac{\delta\rho V_A^A(x_n)}{1-\delta(1-\rho)}$. Then we will show that $V_A^A(x_{n+1}) > x_{n+1} + \frac{\delta\rho V_A^A(x_{n+1})}{1-\delta(1-\rho)}$.

On the equilibrium path,

$$V_A^A(x_{n+1}) = \delta\left(\rho V_A^A(x_n) + (1-\rho)\frac{\delta\rho V_A^A(x_n)}{1-\delta(1-\rho)}\right) \tag{32}$$

$$= \frac{\delta\rho V_A^A(x_n)}{1-\delta(1-\rho)} \tag{33}$$

We also know (from Lemma 2) that $x_{n+1} = \frac{\delta\rho x_n}{1-\delta(1-\rho)}$. We can re-write our assumed condition as

$$\frac{\delta\rho}{1-\delta(1-\rho)}V_A^A(x_n) > \left(x_n + \frac{\delta\rho V_A^A(x_n)}{1-\delta(1-\rho)}\right)\frac{\delta\rho}{1-\delta(1-\rho)} \tag{34}$$

(35)

$$V_A^A(x_{n+1}) > \frac{\delta\rho x_n}{1-\delta(1-\rho)} + V_A^A(x_{n+1})\frac{\delta\rho}{1-\delta(1-\rho)}$$

$$V_A^A(x_{n+1}) > x_{n+1} + V_A^A(x_{n+1})\frac{\delta\rho}{1-\delta(1-\rho)} \tag{36}$$

This is the condition we set out to show. □

Proof of Proposition 2: When the status quo policy equals 0 (favorable to player B), we will show that it is never the case that two conditions hold: (i) A accepts a retraction of authority, and (ii) B wants to retract authority. It is convenient in this proof to treat A and B's expected utilities (denoted $V_A(x_n)$ and $V_B(x_n)$, respectively) as portions of the total pie. Since we know that $V_A(x_n) + V_B(x_n) = \frac{1}{1-\delta} \Rightarrow (1-\delta)V_A(x_n) + (1-\delta)V_B(x_n) = 1$. We will let a and b represent two possible values of A's expected utility as a portion of 1, with $a < b$.

Recall that $x_n = \frac{\delta^{n-1}(x_1(1-\delta)+\delta(1-\epsilon))\epsilon\rho^{n-1}}{(1-\delta(1-\rho))^{n-1}(1-\delta-\delta(1-\epsilon)\rho)}$ and $V_A(x_n) = \frac{\delta^{n-1}(x_1(1-\delta)+\delta(1-\epsilon))\rho^{n}}{(1-\delta)(1-\delta(1-\rho))^{n-1}(1-\delta-\delta(1-\epsilon)\rho)}$. We can rewrite x_n as $\frac{\epsilon}{\rho}b$.

First, the condition for A to accept retraction is

$$1 \geq \frac{\epsilon}{\rho}b + \epsilon\frac{\delta}{1-\delta}(b-a) \tag{37}$$

$$\frac{1}{\epsilon} - \frac{b}{\rho} \geq \frac{\delta}{1-\delta}(b-a) \tag{38}$$

Second, the condition for B to want to retract authority rather than accept current policy benefits

$$\frac{\delta}{1-\delta}(1-a) \geq 1 + \frac{\delta}{1-\delta}(1-b) \tag{39}$$

$$\frac{\delta}{1-\delta}(b-a) \geq 1 \tag{40}$$

We can rearrange these inequalities to be $1 \leq \frac{\delta}{1-\delta}(b-a) \leq \frac{1}{\epsilon} - \frac{b}{\rho}$. To guarantee that there is no possible value of a, which can be treated as a choice variable for B (in that B holds proposal power), it must be the case that $\frac{1}{\epsilon} - \frac{b}{\rho} < 1$. For this to hold requires that

$$1 > \frac{1}{\epsilon} - \frac{1}{\rho}\left(\frac{\delta^{n-1}(x_1(1-\delta)+\delta(1-\epsilon))\rho^n}{(1-\delta(1-\rho))^{n-1}(1-\delta+\delta(1-\epsilon)\rho)}\right) \tag{41}$$

$$\frac{\delta^{n-1}(x_1(1-\delta)+\delta(1-\epsilon))\rho^{n-1}}{(1-\delta(1-\rho))^{n-1}(1-\delta+\delta(1-\epsilon)\rho)} > \frac{1}{\epsilon} - 1 \tag{42}$$

$$\frac{\delta^{n-1}\rho^{n-1}}{(1-\delta(1-\rho))^{n-1}} > \frac{\left(\frac{1}{\epsilon}-1\right)(1-\delta+\delta(1-\epsilon)\rho)}{(x_1(1-\delta)+\delta(1-\epsilon))} \tag{43}$$

This condition always holds for $\epsilon > \bar{\epsilon}$, as shown in Lemma 3. □

Proof of Proposition 3: In this proof, we will show that moving authority is a dominant strategy for A whenever the status quo policy is less than or equal to x. In conjunction with the result stated in Proposition 2, that player B chooses $s = 0$ in every state on the equilibrium path when A chooses to move authority in every state on the equilibrium path, and the fact that the status quo policy in the first period is assumed to be favorable to B ($s_0 = 0$), this is sufficient to prove the proposition.

In any period when A is the proposer, A chooses to either move authority or take current policy benefits. There are two cases. Case 1: Assuming A cannot move the game to $x = 1$, the condition for B to accept is

$$1 + \frac{\delta}{1-\delta} a = 1 - x_t + \frac{\delta}{1-\delta}\left(\epsilon b + (1-\epsilon) a\right) \tag{44}$$

This condition holds when $a = \frac{b\delta\epsilon - x(1-\delta)}{\delta\epsilon}$. Given this expression for a, the condition for A to move authority is

$$\frac{\delta}{1-\delta}(1 - \frac{b\delta\epsilon - x(1-\delta)}{\delta\epsilon}) > x_t + \frac{\delta}{1-\delta}\left(1-b\right) \tag{45}$$

And this condition always holds. Hence, A strictly prefers to move authority in the first period, assuming A cannot move the game to x_0.

Case 2: Assuming A can move the game to x_0 in the first period, then the condition for B to accept is

$$\bar{s} = 1 - x_t + \frac{\delta}{1-\delta}\epsilon b \tag{46}$$

The condition for A to offer $\bar{s}$ is

$$1 - \left(1 - x_t + \frac{\delta}{1-\delta}\epsilon b\right) + \frac{\delta}{1-\delta} > x_t + \frac{\delta}{1-\delta}(1-b) \tag{47}$$

This condition also always holds. Hence, A strictly prefers to move authority in any period in which A is the proposer and $s_{t-1} \leq x_t$.

If A is drawn in the first period, then A chooses to move authority because $s_0 = 0 \leq x_t$. In subsequent periods, when B is drawn, by Proposition 2 we know that B will choose to take current policy benefits. Hence, in all subsequent periods, the status quo policy will be $s_{t-1} = 0$, and A will continue to move authority until $x = 1$.

If B is drawn in the first period, then by Proposition 2 we know that B will take current policy benefits, since $s_0 = 0$. This holds for any period in which B is drawn prior to A being drawn for the first time. Once A is drawn as proposer for the first time, then the above argument follows.

Therefore, no matter which player is drawn as proposer in the first period, the developmental equilibrium path results in which A and B choose $s = 0$, while A increments x toward $x = 1$, in every period until $x = 1$. □

B *B*'s expected utility

This appendix proves (i) the value of B's expected utility in every state, and (ii) the sum of A and B's expected utilities on the equilibrium path characterized in Lemma 2 equals $\frac{1}{1-\delta}$.

Using the value of s from Lemma 2, we know that B's expected utility from accepting A's offer in x_1 is simply $\bar{s}$. Prior to A being drawn as the proposer in state x_1, B's expected utility is

$$V_B(x_1) = \rho\left(= 1 - \frac{x_1(1-\delta(1-\rho)) - \delta\epsilon}{1-\delta(1-\rho(1-\epsilon))}\right) + (1-\rho)(1+\delta V_B(x_1)) \tag{48}$$

$$= \frac{1 - x_1\rho}{1-\delta+\delta(1-\epsilon)\rho} \tag{49}$$

The value of x_n from Lemma 2 is $\frac{\delta^{n-1}(x_1(1-\delta)+\delta(1-\epsilon))\epsilon\rho^{n-1}}{(1-\delta(1-\rho))^{n-1}(1-\delta-\delta(1-\epsilon)\rho)}$, which implies that x_{n+1} is $\frac{\delta^{n}(x_1(1-\delta)+\delta(1-\epsilon))\epsilon\rho^{n}}{(1-\delta(1-\rho))^{n}(1-\delta-\delta(1-\epsilon)\rho)}$.

The condition for B to accept A's offer in state x_{n+1} is

$$1 + \delta V_B(x_n) = 1 - x_{n+1} + \delta\left(\epsilon V_B(x_{n+1}) + (1-\epsilon)V_B(x_n)\right) \tag{50}$$

$$V_B(x_{n+1}) = V_B(x_n) + \frac{x_n}{\delta\epsilon} \tag{51}$$

Substituting in the value of x_{n+1} from above, we have

$$V_B(x_{n+1}) = V_B(x_n) + \frac{1}{\delta\epsilon}\frac{\delta^n(x_1(1-\delta)+\delta(1-\epsilon))\epsilon\rho^n}{(1-\delta(1-\rho))^n(1-\delta-\delta(1-\epsilon)\rho)} \tag{52}$$

We can label $\Delta_n := \frac{\delta^{n-1}(x_1(1-\delta)+\delta(1-\epsilon))\rho^n}{(1-\delta(1-\rho))^n(1-\delta-\delta(1-\epsilon)\rho)}$. Thus, we have a value for B's expected utility in every state, which is

$$V_B(x_{n+1}) = V_B(x_n) + \Delta_n \tag{53}$$

We can use a brief induction argument to show that the sum of A and B's expected utilities always equal $\frac{1}{1-\delta}$ on the equilibrium path characterized in Lemma 2. First, we have the base case:

$$\frac{1-x_1\rho}{1-\delta+\delta(1-\epsilon)\rho} + \frac{(x_1+\delta-x_1\delta-\delta\epsilon)\rho}{(1-\delta)(1-\delta+\delta(1-\epsilon)\rho)} = \frac{1}{1-\delta} \tag{54}$$

Second, we assume that A and B's expected utilities in state x_n equal $\frac{1}{1-\delta}$ in order to show that A and B's expected utilities in state x_{n+1} equal $\frac{1}{1-\delta}$. As proven above, B's expected utility in state x_{n+1} equals that of state x_n plus Δ_n. So the condition we want is $V_B(x_n) + \Delta_n + V_A(x_{n+1}) = \frac{1}{1-\delta}$. We can solve for $V_B(x_n)$ from the assumed condition (that

$V_B(x_n) + V_B(x_{n+1}) = \frac{1}{1-\delta}$) to obtain $V_B(x_n) = \frac{1}{1-\delta} - \frac{\delta^{n-1}(x_1(1-\delta)+\delta(1-\epsilon))\rho^n}{(1-\delta)(1-\delta(1-\rho))^{n-1}(1-\delta-\delta(1-\epsilon)\rho)}$. Then we can substitute this to get

(55)
$$\frac{1}{1-\delta} - \frac{\delta^{n-1}(x_1(1-\delta)+\delta(1-\epsilon))\rho^n}{(1-\delta)(1-\delta(1-\rho))^{n-1}(1-\delta-\delta(1-\epsilon)\rho)} + \frac{\delta^{n-1}(x_1(1-\delta)+\delta(1-\epsilon))\rho^n}{(1-\delta(1-\rho))^{n}(1-\delta-\delta(1-\epsilon)\rho)}$$
$$+ \frac{\delta^{n}(x_1(1-\delta)+\delta(1-\epsilon))\rho^{n+1}}{(1-\delta)(1-\delta(1-\rho))^{n}(1-\delta-\delta(1-\epsilon)\rho)} = \frac{1}{1-\delta}$$

This completes the proof by mathematical induction.