INEQUALITY, FISCAL POLICY AND COVID19 RESTRICTIONS IN A DEMAND-DETERMINED ECONOMY

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Abstract: We evaluate the effects of inequality, fiscal policy, and COVID19 restrictions in a model of economic slack with potentially rigid capital operating costs. Inequality has large negative effects on output, while also diminishing the effects of demand-side fiscal stimulus. COVID restrictions can reduce current-period GDP by more than is directly associated with the restrictions themselves when rigid capital costs induce firm exit. Higher inequality is associated with larger restriction multipliers. The effectiveness of fiscal policies depends on inequality and the joint distribution of capital operating costs and firm revenues. Furthermore, COVID19 restrictions can cause future inflation, as households tilt their expenditure toward the future.

Keywords: COVID19, fiscal policy, firm exit, spending multipliers, inequality.

JEL: E62, E32, H3

"*Low-income households have experienced, by far, the sharpest drop in employment, while job losses of African-Americans, Hispanics and women have been greater than that of other groups. If not contained and reversed, the downturn could further widen gaps in economic well-being that the long expansion had made some progress in closing*" –Jerome Powell, testimony to Senate Banking Committee, June 16th, 2020.

1. Introduction

While painful, the economic restrictions associated with the COVID19 pandemic have served as a grand natural experiment with the potential to shed light on fundamental relationships in the economy and to inform us on the design of optimal policies. Specifically, employment and income losses have been concentrated among low-income households and – as suggested by the Fed Chairman Powell – the rise in inequality is a clear and concerning development. However, it is less clear how inequality transmits macroeconomic shocks, including those associated with the pandemic. Relatedly, what is the economic effect of the fiscal policy response to the COVID19 crisis, and how does inequality mitigate or amplify the effects of fiscal policy? While this is a truly $2 trillion question, there is little clarity on how the stimulus works in the current conditions. Indeed, one may think that fiscal policy is more stimulative in recessions[1] but e.g., Brunet (2018) documents evidence suggesting that fiscal multipliers were smaller during World War II because the government imposed restrictions on how households could spend their income.

To shed light on the role of inequality in transmitting macroeconomic shocks, we examine COVID19-related restrictions and fiscal policy in a model of economic slack in which inequality and capital costs play a central role.[2] We extend the negligible-marginal-cost (NMC) framework of Murphy (2017) to examine heterogeneous (rich and poor) households that consume a variety of goods and services. The central feature of the NMC framework is that additional output does not require additional factor inputs, which implies that aggregate demand – rather than aggregate supply – determines aggregate output. Furthermore, inequality and the firm entry margin play a central role in magnifying macroeconomic shocks, which distinguishes the NMC framework from other recent models of firm-level slack (e.g., Michaillat and Saez 2015).

The mechanism through which inequality affects output is a formalization of the relationship conjectured by Krugman (2016) and Summers (2015) that declines in the labor share pull down

[1] See e.g. Auerbach and Gorodnichenko (2012, 2013).

[2] Theories of economic slack posit that workers and capital experience periods of idleness that represent wasted resources (e.g., Michaillat and Saez 2015, Murphy 2017). For empirical evidence of the relevance of models of slack, see e.g. Auerbach et al. (2020a, 2020b), Demyanyk et al. (2019), Egger et al. (2020), and Boehm and Pandali-Nayar (2017).

aggregate demand. Households have non-homothetic preferences over NMC-sector goods/services (i.e., goods/services that have production characterized by negligible marginal costs) and a numeraire good such that the rich satiate their demand for goods/services in the NMC sector (and therefore their per-product demand is invariant to macroeconomic conditions), whereas poor households' spending on NMC goods/services is limited by their income (which in turn depends on spending by the rich and on fiscal transfers). Therefore a lower income share for the poor (and hence higher inequality) is associated with lower spending by poor households and hence lower aggregate income and output. Inequality has strong effects even though all households can access credit.

The model provides a lens through which to understand changes in the spending preferences of rich households, a prominent feature of the pandemic (Chetty et al. 2020). Cuts in rich-household spending are associated with large Keynesian multiplier effects, with the size of the effect increasing in the income share of the poor. The larger is the income share of the poor (the lower is inequality), the more a spending shock circulates back to the poor as additional income (and hence additional spending). Therefore, the model implies that while inequality has had a direct effect of reducing output, it has weakened the (nonetheless strong) effects of rich-household spending cuts.

We model the economic restrictions associated with COVID19 as a temporary decrease in the share of varieties of goods/services that can be exchanged. The effect of COVID19 restrictions depends on the steady-state level of transfers to poor households, as well as the extent to which firms face fixed capital operating costs. If there are no steady-state transfers and capital operating costs are flexible (such that the price of capital adjusts to prevent firm exit), then the output loss is proportional to the fraction of varieties that are directly subject to COVID19 restrictions – that is, the restriction multiplier is unity, and consumption of unrestricted products remains the same. This is because the direct reduction in household spending on restricted products equals the reduction of household income from selling those products (and hence no additional adjustments are necessary).

Positive steady-state transfers to the poor lead to a smaller restriction multiplier. Households allocate their expenditure across varieties *and* across time. When fewer varieties are available in the current period, households reallocate their transfer income (if any) toward the remaining existing products during the pandemic and toward products available in the future. This higher per-product spending leads to higher output per product in the current period, which mitigates the aggregate output effects of the restrictions. The model also predicts that the restrictions cause future nominal spending to increase beyond what it would have been in the absence of restrictions (due to the expenditure reallocation across time). If prices are flexible in the future, the restrictions cause future inflation even in the absence of additional government stimulus. However, inequality dampens this channel, since transfers multiply into output less when poor households earn less of each dollar spent. Therefore, the model predicts a rise in future inflation, albeit less than what would have occurred in the absence of rising inequality.

Restriction multipliers are much larger when firms' fixed operating costs are rigid. Restrictions on a subset of firms' products pulls down firm revenue, which causes firms for which fixed operating costs are high relative to steady-state revenues to exit and therefore cease production of other unrestricted goods and services. For example, restaurants are restricted from serving customers in the establishment but are able to provide carry-out and delivery services, and airlines shut down some routes (and/or passenger seats) but maintain others. If restrictions cause some restaurants' revenues to decline below their fixed costs, then these restaurants will cease producing carry-out services. Likewise, airlines with revenues below fixed costs will cease flying entirely. This firm exit channel leads to large indirect (multiplier) effects of economic restrictions and provides a strong rationale for policies aimed at mitigating fixed operating costs. In the absence of these multiplier effects or significant re-entry costs, it might be optimal to allow firms to temporarily exit and then re-enter once restrictions are lifted. But the large multipliers imply that such exit can be very costly.

We examine the effect of fiscal transfers in this environment. Government transfers to low-income households have multiplier effects, which can offset the adverse secondary (multiplier) effects of the COVID19 restrictions. However, the transfers can have smaller multiplier effects during the presence of COVID19 restrictions, since there are fewer products on which to spend. Transfers also increase low-income households' total spending capacity, which leads to a larger nominal spending (inflation) boom after restrictions are lifted.

All fiscal policy is not equal, however. In our framework the government has various fiscal levers that it can pull: direct transfers to households, direct transfers to firms, as well as various targeted transfers. The preferred policy depends on inequality, the joint distribution of firm revenues and fixed operating costs, and the extent to which the government can target households and firms.

Targeted transfers to low-income households can increase spending on unrestricted items, thus supporting income during the restrictions. However, the transfers have stronger effects on expenditure in the future. Furthermore, the output effect of transfers is falling in inequality, as spending multipliers are increasing in the income share of the poor. One of the prominent features of the COVID recession has been an increase in inequality. Our model predicts that transfers can help offset the adverse effects of rising inequality. But the transfers are also less effective as stimulus the higher is inequality, as less of the initial spending induced by the transfers circulates back as income for the poor (and hence leads to less additional spending).

The strongest effect of fiscal stimulus is through targeted transfers to multiproduct firms for which the restrictions push their revenues below their fixed operating costs. Such targeted transfers prevent firm exits that lead to large secondary (multiplier) output declines. In practice it may be difficult to identify and target such firms, although the model offers some guidance. The firms most at risk of exit are those with relatively low profitability and for which fixed operating costs are the largest or most rigid. As documented by Gilje et al. (2020), rigid capital contracts can arise from asymmetric information regarding firms' ability to cover capital costs. In our context, the asymmetric information friction is perhaps the most severe for smaller businesses that are not subject to the same reporting requirements as public firms. Direct loans and transfers to small private businesses may therefore target the firms on the margin of exit and have large benefits per dollar spent.

While targeted transfers to firms have the largest potential benefit, untargeted transfers to firms have among the least benefit. Not only is some income spent on firms that are not in danger of exit, but a large share of the income received by the firms accrues to high-income households for whom spending is less sensitive to transfers.

This paper is broadly related to emerging work evaluating the indirect economic effects of COVID19, with different papers focusing on different transmission channels. For example, Baqaee and Farhi (2020) focus on the production network, Fornaro and Wolf (2020) focus on productivity growth, and Caballero and Simsek (2020) examine the role of asset prices. Using a large calibrated HANK model, Auclert and Rognlie (2020) find that, for standard business cycles, inequality has relatively mild effects on output, while we find that the effect of inequality can be quite large in the NMC setting.[3] Most closely related is Guerrieri et al. (2020), who model COVID19 as a restriction on labor supplied to a subset of firms. They argue that COVID19 restrictions can cause a further fall in output in the presence of strong complementarities between restricted goods and other goods

[3] While Auclert and Rognlie (2020) focus primarily on the role of income risk in a New Keynesian framework, we abstract from income risk and focus on permanent earnings differences between rich and poor households.

(elasticity of substitution across products, EOS), a large intertemporal elasticity of substitution (IES), and large shares of credit-constrained households–parameters for which there is strong disagreement in the literature. If these conditions are sufficiently strong, the economy can exhibit a multiplier whereby output falls by more than the size of the direct supply restrictions. Our approach to modeling the COVID19 restrictions is similar in that a subset of firms cannot sell output to consumers but the transmission mechanisms in our model are quite different (e.g., our model does not rely on credit constraints, high IES, or low EOS).[4] A distinguishing feature of our analysis is that we examine these restrictions in a model environment in which inequality, fixed operating costs, and multiproduct firms play a central role. We find that output effects of the restrictions are potentially large even if credit is unrestricted. We furthermore show that large output effects are limited to the direct effects of restrictions on a subset of goods and services unless firms face fixed capital operating costs. Finally, we evaluate the benefits of alternative fiscal stimulus measures, including (targeted and untargeted) transfers to households and firms. The relative effectiveness of alternative fiscal stimulus measures depends on a number of conditions, including inequality and the joint distribution of firms' revenues and capital costs. In this sense our framework can guide empirical work examining the relative merits of alternative stimulus measures. More generally, our model sheds light on the potentially large adverse effects of inequality on GDP.

2. Baseline Model

To study fiscal policy, we develop the heterogeneous-household version of the negligible-marginal-cost (NMC) model in Murphy (2017). This version of the model features rich and poor households, denoted by $h \in \{\mathbb{R}, \mathbb{P}\}$, each of which receives different shares of income from the NMC sector and consumes services from the NMC sector. The model also features an endowment that is owned and consumed by the rich. The endowment represents land or other factors of production that are used to produce goods consumed primarily by the rich (e.g., beach homes and other luxury items). The endowment pins down the interest rate and the consumption path of the

[4] The firm exit margin is the driving force behind large effects of COVID restrictions in our model. In Guerrieri et al. (2020), firm exit amplifies the effects of restrictions operating through credit constraints, a low EOS, and a high EIS. In other words, in their model the consumer spending channel is necessary for the firm exit margin to matter, whereas in our model the firm exit margin is driven by multiproduct firms and does not rely on consumer spending multipliers.

rich household. Agents trade bonds to satisfy their desired time paths of consumption, subject to a no-Ponzi constraint that the present value of their asset position must be weakly greater than zero.

We assume that time can be split into two periods: $t = 0$ which captures the crisis and $t = 1$ which correspond to the post-crisis time. We evaluate policy responses to a one-time restriction in spending at date $t = 0$. Without loss of generality, we subsume all future periods into a single date $t = 1$.

Households. There is a unit mass of homogenous varieties in the NMC sector. Households inelastically supply labor to the NMC sector, and there are zero marginal costs of labor associated with increasing output.[5] In this sense there is firm-level slack. In the initial period, a share $1 - \xi$ of the varieties is restricted from being sold. We interpreted a reduction in ξ as either direct restrictions imposed by the government or choices by households to avoid certain services due to health concerns.[6]

Household type h maximizes

$$U^h = \sum_{t=0}^{1} \beta^t \left(y_t^h + \int_0^{\psi_t} \int_0^{\xi_t} \left(\theta^h q_{jkt}^h - \frac{\gamma}{2} \left(q_{jkt}^h \right)^2 \right) dk\, dj \right), \tag{1}$$

subject to the budget constraints

$$\int_0^{\psi_0} \int_0^{\xi_0} p_{jk0} q_{jk0}^h dkdj + y_0^h + QB = \Pi_0^h + e_0^h + T_0^h, \tag{2}$$

$$\int_0^{\psi_1} \int_0^{\xi_1} p_{jk1} q_{jk1}^h dkdj + y_1^h = \Pi_1^h + e_1^h + T_1^h + B, \tag{3}$$

where q_{jkt}^h is type h's consumption of variety $k \in [0,1]$ from firm $j \in [0,1]$ from the NMC sector in period t. The household's preferences are over each producer-commodity (jk) element. ξ_t is the fraction of goods/services that can be sold without restriction and $\psi_t \leq 1$ is the endogenously determined number of firms in the economy. We will assume that $\xi_t = 1$ and $\psi_t = 1$ in the absence of COVID-related restrictions, and that the restrictions imply $\xi_0 \equiv \xi < 1, \xi_1 = 1$ (and

[5] See Auerbach et al. (2020b) for an overview of the empirical relevance of negligible marginal labor costs.
[6] See Alexander and Karber (2020), Chetty et al. (2020), and Goolsbee and Syverson (2020) for evidence that households voluntarily avoided purchases of services perceived to be high-risk.

potentially $\psi_t < 1$).[7] For simplicity we assume that reductions in ξ are associated with equal restrictions for poor and rich households, although there is some evidence that rich households may have been more likely to avoid spending due to health concerns (Chetty et al. 2020).[8] Π_t^h is agent h's income from the NMC sector of the economy, e_t^h and y_t^h are h's endowment and consumption of the numeraire, where $e_t^{\mathbb{P}} = 0$. T_t^h is net transfers from the government. Q is the price of a bond B that pays a unit of the numeraire in period 1. Since agents can smooth consumption (and hence the effect of the present value of future net transfers is the same as the effect of present-period net transfers), we will write the present value of total net transfers as $T^h \equiv T_0^h + QT_1^h$.

A convenient feature of the quasilinear utility function is that agents consume only the good from the NMC sector when their income is sufficiently low (depending on θ and γ).[9] This feature, along with the assumption that poor agents are not endowed with the numeraire, $e_t^{\mathbb{P}} = 0\ \forall\ t$, simplifies the analysis and maintains the focus on demand-determined output in the NMC sector. We assume parameter values such that only the rich household consumes the numeraire endowment good. One implication of this assumption is that, similar to the Lucas-tree model, variation in endowments e pins down Q to the discount factor β of the rich, that is $Q = \beta$. This assumption is a reduced-form attempt to model the economy when interest rates are fixed at some level (for example, the effective lower bound).

Firms. Output in the NMC sector is produced by firms who hire workers as fixed costs and pay a fixed capital operating cost f_{jt}. Firm j faces demand for product jk from household type h

$$q_{jkt}^h = \frac{1}{\gamma}\left(\theta^h - \lambda_{ht} p_{jkt}^h\right), \qquad (4)$$

where λ_{ht} is household h's budget multiplier at time t.

We assume that poor-household prices for NMC goods/services in period $t = 0$ are fixed at the levels $\bar{p}_{jk0}^{\mathbb{P}}$ that we would observe if firms set their prices on their expectation that $\xi = 1$.

[7] Given the separability of preferences, shutting down access to any jkt element has symmetric effects on all other jkt elements and hence there are no changes in the composition of remaining commodities (and hence no *direct* demand spillover effects on unaffected producer-commodities).

[8] As discussed below, spending adjustments by the rich only map into reductions in $\theta^{\mathbb{R}}$ (and hence $c_{jk0}^{\mathbb{R}}$).

[9] While the rich households' marginal propensity to consume (MPC) on NMC goods/services is zero, their MPC that includes spending on the endowment e is equal to the poor households' MPC on NMC goods/services (the poor do not spend anything on endowment good e). Hence, the "total" MPC is the same for the poor and the rich.

This price remains fixed in the presence of shocks in period $t = 0$. We write $\bar{p}_{jk0}^{\mathbb{P}}$ with an overbar to emphasize that these prices are rigid. The degree to which prices in period $t = 0$ are rigid determines the extent to which shocks affect real GDP. With rigid prices, real GDP is more responsive to shocks. Prices in the post-crisis period $t = 1$ are fully flexible.

For analytic convenience, we assume that firms can price discriminate between the rich and the poor. The profit-maximizing price charged to household type h is

$$p_{jkt}^{h} = \frac{\theta^{h}}{2\lambda_{ht}^{S}}, \tag{5}$$

where due to rigid initial-period prices λ_{h0}^{S} is the household h's period-0 budget multiplier in the state of the world in which there are no shocks ($\xi = 1$) and $\lambda_{h1}^{S}$is the household's period-1 budget multiplier adjusted for the realization of shocks. The rich household's budget multiplier is pinned down by marginal utility of the numeraire, $\lambda_{\mathbb{R}t} = 1$, so prices charged to the rich are invariant to all shocks other than the rich household's preference for NMC-sector goods/services $\theta^{\mathbb{R}}$.

Given prices in equation (5) and imposing $\lambda_{\mathbb{R}t} = 1$ we can write quantities demanded as

$$q_{jk0}^{\mathbb{P}} = \frac{1}{\gamma}\left(\theta^{\mathbb{P}} - \lambda_{h0}\bar{p}_{jk0}^{\mathbb{P}}\right), \qquad q_{jk1}^{\mathbb{P}} = \frac{\theta^{\mathbb{P}}}{2\gamma}, \qquad q_{jk0}^{\mathbb{R}} = q_{jk1}^{\mathbb{R}} = \frac{\theta^{\mathbb{R}}}{2\gamma}, \tag{6}$$

and expenditure by each household type h on each good jk as

$$c_{jkt}^{h} \equiv p_{jkt}^{h} q_{jkt}^{h}. \tag{7}$$

For rich households, we can write $c_{jkt}^{\mathbb{R}} = \left(\theta^{\mathbb{R}}\right)^{2}/4\gamma$. Rich-household expenditure on any given firm-commodity is a function only of exogenous parameters and we therefore treat $c_{jkt}^{\mathbb{R}}$ as exogenous for the remainder of the analysis. This invariance of rich-household expenditure to macroeconomic conditions greatly simplifies the analytic derivation of results. One can interpret $c_{jkt}^{\mathbb{R}}$ as "autonomous" spending in the economy.

A firm j's revenues are equal to expenditure across households and products: $R_{jt} = \int_{0}^{\xi_t}\left(c_{jkt}^{\mathbb{R}} + c_{jkt}^{\mathbb{P}}\right)dk$. By symmetry of varieties (all firm-commodity combinations that continue to be produced in equilibrium have the same revenue), we can write $R_{jt} = \xi_t\left(c_{jkt}^{\mathbb{R}} + c_{jkt}^{\mathbb{P}}\right)$. Firm j pays a fixed capital operating cost f_{jt} in period t.[10] We assume that households own capital in the

[10] We assume that cost f_{jt} is fixed in nominal terms in period $t = 0$ but it is free to adjust in period $t = 1$ so that the mass of firms cannot be greater than 1.

same proportion to their share of firm profits and so we roll capital income into profits (i.e., Π includes profits and f_{jt}). A firm exits for period t if $R_{jt} < f_{jt}$. We assume that the distribution of fixed costs is such that the unit mass of firms all produce if $\xi = 1$ and that only a share $\psi_0(\xi) < 1$ continue to produce in the initial period if $\xi < 1$. If there are additional costs to re-entry once restrictions are lifted, then $\psi_1 < 1$. In the absence of such costs to re-entry, $\psi_1 = 1$.

The poor household receives a share κ of the revenues from the NMC sector in each period, while the rich household receives the remaining $1-\kappa$ share. The poor household also owns a share κ of the capital stock (and therefore earns a share κ of the payments from firms for fixed capital operating costs). It can be shown that there exists a threshold value $\bar{\kappa}$ such that $\forall\ 0 < \kappa < \bar{\kappa}$, the poor consume output only from the NMC sector. $\bar{\kappa}$ depends on model parameters and fiscal policy. We assume parameter values such that $\kappa < \bar{\kappa}$.

Output (real GDP) Y_t is defined as the product of quantities consumed per product and total mass of available products:

$$Y_0 = \xi\psi_0\left(q_{jk0}^{\mathbb{P}} + q_{jk0}^{\mathbb{R}}\right), \qquad Y_1 = \psi_1\left(q_{jk1}^{\mathbb{P}} + q_{jk1}^{\mathbb{R}}\right). \tag{8}$$

Equilibrium. Equilibrium consists of prices and quantities such that households maximize utility (1) subject to budget constraints (2) and (3), firms' prices are given by equation (5), and ψ_t is determined by the number of firms for which revenues exceed fixed capital operating costs (specified below).[11]

The interesting aspects of the equilibrium are based on the expenditure of poor households (since the rich household's expenditure is effectively exogenous). Total expenditure by household h in period t is the sum of expenditure on the varieties. Given the assumptions about ξ_t, we can write

$$c_0^h = \int_0^{\psi_0}\int_0^{\xi_0} c_{jk0}^h dk\, dj \ = \psi_0\xi c_{jk0}^h, \quad c_1^h = \int_0^{\psi_1}\int_0^{\xi_1} c_{jk1}^h dk\, dj \ = \psi_1 c_{jk1}^h. \tag{9}$$

[11] Note that in general, the market for the endowment good clears even with taxes and transfers and no change in its price. For example, when the government taxes the endowment of the rich, the taxed portion eventually ends back in the hands of the rich as poor households spend the transfer on the NMC sector. If a poor household is given a dollar in transfers, it will spend the dollar on NMC goods/services. $1-\kappa$ share of the dollar will become income of the rich (who will spend it on the endowment good) while κ share will become income of the poor. This "second-round" income of the poor will be spent on the NMC goods/services again so that $(1-\kappa)\kappa$ will become income of the rich and κ^2 will become income of the poor. These rounds of spending will continue and, in the end, the rich will get their \$1dollar in taxes back in income $(1-\kappa) + (1-\kappa)\kappa + (1-\kappa)\kappa^2 + \cdots = 1$ which they spend on the endowment good.

Let $C^{\mathbb{P}}$ be the present value of the poor household's total lifetime expenditure. Then substituting (9) into (2) and (3) and simplifying implies that the present value of the poor household's total lifetime expenditure is

$$C^{\mathbb{P}} = c_0^{\mathbb{P}} + Qc_1^{\mathbb{P}} = \psi_0 \xi c_{jk0}^{\mathbb{P}} + Q\psi_1 c_{jk1}^{\mathbb{P}}. \tag{10}$$

To be clear, c_{jkt}^h represents the equilibrium level of spending, which is the same for any jk produced in equilibrium. To save notation, from now on, jk denotes spending on any variety. The poor household's lifetime income $I^{\mathbb{P}}$ is

$$\begin{aligned} I^{\mathbb{P}} &= \kappa\psi_0 R_{j0} + Q\kappa\psi_1 R_{j0} + T^{\mathbb{P}} \\ &= \kappa\psi_0 \xi\left(c_{jk0}^{\mathbb{P}} + c_{jk0}^{\mathbb{R}}\right) + Q\left(\kappa\psi_1\left(c_{jk1}^{\mathbb{P}} + c_{jk1}^{\mathbb{R}}\right)\right) + T^{\mathbb{P}}, \end{aligned} \tag{11}$$

which reflects the fact that the poor household earns a share κ of total expenditure. Since households own capital in the same proportion to their share of firm profits and households (as firm owners) are both liable for firms' capital operating costs and receive income from payments to capital, capital costs and income are netted out of household income.

Setting lifetime expenditure $C^{\mathbb{P}}$ equal to lifetime income $I^{\mathbb{P}}$ and collecting terms yields

$$c_{jk0}^{\mathbb{P}}\psi_0\xi(1-\kappa) + c_{jk1}^{\mathbb{P}}\psi_1(Q - Q\kappa) = \kappa c_{jk0}^{\mathbb{R}}(\psi_0\xi + Q\psi_1) + T^{\mathbb{P}}, \tag{12}$$

where we have substituted $c_{jk0}^{\mathbb{R}} = c_{jk1}^{\mathbb{R}}$ based on the rich household's expression for expenditure and its first-order conditions with respect to the numeraire and the bond.

NK vs. NMC frameworks.

To draw contrast between the NMC framework and the mainstream New Keynesian (NK) approach, note that a simple way of capturing the mechanics of a New Keynesian model is to assume

$$Y_0^{NK} = C_0, \qquad Y_1^{NK} = \bar{Y}, \tag{13}$$

where the superscript indicates the New Keynesian representation of the model. Here, future output Y_1 is determined by the endowment $\bar{Y}$, reflecting the supply-side dominance of New Keynesian models at horizons after which price rigidities have dissipated. To solve the model, one must simply determine C_0, which in general will be based on consumption smoothing and an intertemporal budget constraint. A simple version of consumption smoothing can be written as

$$C_0 = C_1, \tag{14}$$

and the budget constraint can be written (assuming $\beta = 1$) as

$$C_0 + C_1 = Y_0^{NK} + Y_1^{NK}, \tag{15}$$

Substituting the equilibrium conditions from (13) and solving for C_0 yields

$$C_0 = \bar{Y} \Rightarrow Y_0 = \bar{Y}, \tag{16}$$

Therefore, in the presence of consumption smoothing (the absence of credit constraints), output in the demand-determined period depends on the future supply side of the economy. In short, in the absence of credit constraints, the supply side dominates. As a result, credit constraints (and associated high MPCs) and the strength of intertemporal substitution are key considerations for policymakers in thinking about the macroeconomic effect of the restrictions (e.g., Guerrieri et al. 2020). If policymakers are persuaded by recent evidence that many low-income households are not credit-constrained but rather have low MPCs (see, e.g., Miranda-Pinto et al. (2020a) for a survey), or if they are persuaded by evidence that the elasticity of intertemporal substitution is well below unity (e.g., Cashin and Unayama 2016; Schmidt and Toda 2019), then they may conclude that output effects of the restrictions are not a reason for policy intervention.

Now consider a situation in which future output is demand-determined.[12] In this case, the equilibrium conditions can be written as

$$C_0 = \frac{1}{2}(Y_0 + Y_1), \tag{17}$$

where $Y_0 = C_0$ and $Y_1 = C_0$ (by consumption smoothing). Here, any level of desired consumption is a potential equilibrium. This is similar to the indeterminacy of equilibria in some NK models featuring liquidity traps (e.g., Benhabib et al. 2002). Our NMC model avoids this indeterminacy problem because a share of consumption (in particular, that of rich households on the NMC sector) is determined by exogenous parameters and is independent of income.

3. Demand Shocks and Fiscal Policy in the NMC model

To study properties of the model described in the previous section, we linearize the model around the steady state with no shocks (i.e., $\xi = 1$, $\psi_0 = \psi_1 = 1$) and no transfers to the poor (i.e., $T^{\mathbb{P}} = 0$). For some exercises, it will be instructive to consider cases where $T^{\mathbb{P}} > 0$ in the steady state. As a first step, we explore how structural parameters such as the share of income going to the poor (which also controls the level of inequality in the economy), spending by the rich ("autonomous" spending), and transfers to the poor affect key endogenous variables in the model. Then we

[12] In the NMC model, households smooth over changes in expenditure (to a first-order approximation) and future nominal expenditure is demand-determined (since in equilibrium future output is pinned down by demand parameters).

introduce the COVID19 shock to the model and investigate how this shock propagates in the economy. Finally, we study how various fiscal policies can counter the COVID19 shock.

The effect of different transfers depends on how they are financed. It is clear that taxing low-income households (which decreases $T^{\mathbb{P}}$) will reduce GDP (all else equal). An alternative source of funding is to exclusively tax the rich. As long as the rich maintain enough post-tax consumption of the numeraire, there will be no effect of this taxation on GDP in the NMC sector for either period. There is also the possibility that the transfers could be money financed through the central bank (e.g., Galí 2019) if, for example, one interprets the numeraire as money (which the government can print) or more generally if the government has a technology to create the numeraire. Interpreting the numeraire as money is consistent with models of monetary non-neutrality driven by money in the utility function. In our model money-financed transfers would have the same effect as taxing the rich. For the remainder of the analysis we assume that transfers are financed either through taxing the rich or through printing money, and we will examine the relative effectiveness of different types of spending.[13]

3.1. Inequality, rich-household spending, and government transfers.

Inspection of equation (12) implies that poor-household expenditure (and hence aggregate expenditure) is falling in inequality and increasing in spending by rich households and in transfer income. In general, any factor that increases the income of the poor generates an increase in spending and aggregate output, as output is limited only by poor households' spending (which is limited by their income):

Proposition 1: GDP and poor-household expenditure are increasing in the income share of the poor (falling in inequality), spending by the rich, and transfers. The effects of rich-household spending and transfers are increasing in the income share of the poor. In particular, in the absence of steady-state transfers $T^{\mathbb{P}}$:

[13] It might seem that an alternative policy is for the government to lend to poor households. However, since the households in this environment are already able to smooth their consumption, the lending has no effect. Therefore, one can think of our model as an environment in which monetary policy has extended credit to households to an extent that is sufficient for them to smooth consumption. The benefits of fiscal transfers are evaluated above and beyond the credit-enhancing benefits of monetary policy.

$$\left.\frac{dc^{\mathbb{P}}_{jk0}}{d\kappa}\right|_{T^{\mathbb{P}}=0}=\frac{c^{\mathbb{P}}_{jk0}+c^{\mathbb{R}}_{jk0}}{1-\kappa}\Rightarrow\left.\frac{dq^{\mathbb{P}}_{jk0}}{d\kappa}\right|_{T^{\mathbb{P}}=0}=\frac{1}{\bar{p}^{\mathbb{P}}_{jk0}}\frac{c^{\mathbb{P}}_{jk0}+c^{\mathbb{R}}_{jk0}}{(1-\kappa)}$$

$$\left.\frac{dc^{\mathbb{P}}_{jk0}}{dc^{\mathbb{R}}_{jk0}}\right|_{T^{\mathbb{P}}=0}=\frac{\kappa}{1-\kappa}\Rightarrow\left.\frac{dq^{\mathbb{P}}_{jk0}}{dc^{\mathbb{R}}_{jk0}}\right|_{T^{\mathbb{P}}=0}=\frac{1}{\bar{p}^{\mathbb{P}}_{jk0}}\frac{\kappa}{1-\kappa} \tag{18}$$

$$\left.\frac{dc^{\mathbb{P}}_{jk0}}{dT^{\mathbb{P}}}\right|_{T^{\mathbb{P}}=0}=\frac{1}{(1+Q)(1-\kappa)}\Rightarrow\left.\frac{dq^{\mathbb{P}}_{jk0}}{dT^{\mathbb{P}}}\right|_{T^{\mathbb{P}}=0}=\frac{1}{\bar{p}^{\mathbb{P}}_{jk0}}\frac{1}{(1+Q)(1-\kappa)}.$$

Proof: Appendix.

As the income share of the poor κ increases (inequality falls), poor households spend more in both periods. Due to rigid initial-period prices, this additional spending translates into higher initial-period consumption by the poor (and hence higher real GDP). The effect of inequality is quantitatively large. One can show that $\left.\frac{d\log c^{\mathbb{P}}_{jk0}}{d\kappa}\right|_{T^{\mathbb{P}}=0}=\frac{1}{\kappa(1-\kappa)}$, which is bounded below by $\frac{1}{0.5(1-0.5)}=4$.

Rich-household spending and transfers from the government also increase income for the poor, which in turn induces higher spending by the poor and higher GDP. These relationships are consistent with recent evidence from Chetty et al. (2020). They document that fiscal transfers associated with the CARES Act increased spending for low-income households. Furthermore, spending cuts during the pandemic were largest among low-income households working in areas that were most exposed to the decline in rich-household spending.

The effect of transfers on GDP is higher the larger is the income share of the poor (the lower is inequality). The relationship between the fiscal transfer multiplier and inequality reflects the fact that the general-equilibrium effects implied by the model are much larger than would be implied by examining the partial equilibrium response of poor-household spending alone, since in general equilibrium the initial spending causes additional poor-household income, which generates additional spending, and so on. For example, the partial-equilibrium effect of transfers on poor-household income is $\frac{\partial I^{\mathbb{P}}}{\partial T^{\mathbb{P}}}=1$. But the general-equilibrium effect – accounting for the effect of poor-household spending on their own income – is $\frac{dI^{\mathbb{P}}}{dT^{\mathbb{P}}}=1+\kappa\frac{dC^{\mathbb{P}}}{dT^{\mathbb{P}}}$, which is rising in the income share of the poor. We discuss this relationship in more detail below when we compare alternative fiscal policies.

3.2. COVID19 shock

The social distancing restrictions associated with COVID19 can be modeled as a decrease in ξ from an initial value of 1, reflecting the restrictions on the exchange of services such as restaurant meals, movie theaters, and sporting events. The size of the restriction multiplier – the net effect of restrictions ($dY_0/d\xi$) relative to the direct effect ($\partial Y_0/\partial\xi$) – depends on the extent to which restrictions cause firms to exit. In the absence of firm exit (e.g., due to flexible capital costs), the restriction multiplier is bounded by unity.

Proposition 2: In the absence of a firm exit margin, the decline in output is bounded by the share of products that are restricted (the restriction multiplier is unity). Firm exit causes a larger fall in output – a restriction multiplier greater than unity:

$$\left.\frac{dY_0}{d\xi}\right|_{T^{\mathbb{P}}=0} = Y_0\left(1+\frac{d\psi_0}{d\xi}\right), \qquad \frac{\partial Y_0}{\partial \xi} = Y_0 \tag{19}$$

Proof: Appendix.

In the absence of transfers, restrictions decrease poor-household aggregate spending to a degree that perfectly balances the decrease in poor-household aggregate income. Given product symmetry, there is no adjustment within product categories.[14] This also implies that in the absence of a firm-entry margin, there are no multiplier effects from product-level restrictions. In other words, GDP falls by an amount proportional to the share of products that are restricted, $\frac{\partial Y_0}{\partial \xi} = Y_0$. If restrictions force firms to exit ($\frac{d\psi_0}{d\xi} > 0$), the restriction multiplier is $\left.\frac{dY_0}{d\xi}\Big/\frac{\partial Y_0}{\partial \xi}\right|_{T^{\mathbb{P}}=0} = 1 + \frac{d\psi_0}{d\xi} > 1$. Chetty et al. (2020) document higher rates of small business closure in places in which spending was cut the most, which suggests that $\frac{d\psi_0}{d\xi} > 0$. Workers in these locations experienced larger income declines and cut their spending by more, consistent with model's prediction of the adverse effect of firm exit on consumer income and spending.

The multiplier effects of COVID restrictions are smaller in the presence of positive steady-state transfers. This is because, in the presence of positive transfers, the poor household responds

[14] There would be within-product adjustments if ξ varied by household. For example, Chetty et al. (2020) document that rich households were more likely to avoid purchasing goods/services with a high risk of infection. In our model, such spending adjustments by the rich only map into reductions in $\theta^{\mathbb{R}}$ (and hence $c_{jk0}^{\mathbb{R}}$).

to the restrictions by spreading its transfer wealth over the fewer available initial-period products and the products available in the future. This per-product spending increase is associated with higher output per product in the initial period (when prices are fixed) and higher prices in the future period (when prices are flexible). We formalize this point in the following proposition.

Proposition 3: (Economic Restrictions and Future Inflation). In the presence of positive steady-state transfers, spending restrictions cause future inflation, as households reallocate spending across the remaining set of goods/services available in the current and future periods:

$$\frac{dc_{jk0}^{\mathbb{P}}}{d\xi} = \frac{dc_{jk1}^{\mathbb{P}}}{d\xi} = -\frac{T^{\mathbb{P}}\left(1+\frac{d\psi_0}{d\xi}\right)}{(1+Q)^2(1-\kappa)}, \qquad \frac{dp_{jk1}^{\mathbb{P}}}{d\xi} = \frac{2}{\theta^{\mathbb{P}}}\frac{dc_{jk0}^{\mathbb{P}}}{d\xi}. \tag{20}$$

Proof: Appendix.

Intuitively, with positive transfers, the poor households' reduction in income is proportionally smaller than the reduction in the number of commodities they buy, so it increases their demand for all remaining commodities. When households must forgo spending on a subset of products, they reallocate their wealth (transfers) across the remaining available products in the initial period *and in the future*. The increase in per-product expenditure causes higher per-product output in the current period (when prices are rigid) and higher prices in the future (when prices are fully flexible). If we were to expand the model to include an intermediate period $t = \{0 \rightarrow 1\}$ in which prices are only partially flexible, then the initial-period restrictions would be followed by a boom in output, as households would consume more per product and (if $d\psi_{\{0\rightarrow 1\}} = 0$) would have the full set of products available to purchase. This prediction of the model is consistent with the behavior of the U.S. economy following World War II. Inflation surged to nearly 20 percent within a year-and-a-half of the end of the war, consistent with households transferring spending power from during the war (when spending was restricted) to after the war.

Note that equation (20) implies that restrictions increase per-product spending (initial-period output and future-period prices) by more the greater the income share of the poor κ (lower inequality), holding fixed the number of firms in the economy. This is because the poor household recycles its purchasing power more the higher is its income share. COVID restrictions increase per-product spending on nonrestricted products (e.g., streaming video services, computer games), and this increased spending is multiplied more the larger is the income share of the poor. Therefore, economic restrictions reduce aggregate output by less (more) in the presence of higher (lower) poor-household income shares, which correspond to lower (higher) inequality. Even accounting for the endogenous response of firm entry, the output response to COVID restrictions can be shown to be increasing in inequality.

Proposition 4: In the presence of steady-state transfers, inequality is associated with stronger output effects of COVID restrictions.

$$\frac{d^2 q_{jk0}^{\mathbb{P}}}{d\xi d\kappa} = -\frac{T^{\mathbb{P}}}{p_{jk0}^{\mathbb{P}}}\left[\frac{(1-\kappa)\frac{d^2\psi_0}{d\xi d\kappa} + \left(1 + \frac{d\psi_0}{d\xi}\right)}{(1+Q)^2(1-\kappa)^2}\right],$$

which is strictly negative as $(1-\kappa)\frac{d^2\psi_0}{d\xi d\kappa} + \left(1 + \frac{d\psi_0}{d\xi}\right) > 0$.

Proof: Appendix.

3.3. The Response of Firm Exit.

The discussion above makes clear that COVID restrictions have large multipliers (>1) in our model only if firms exit. To study this margin, we must specify f_j, the cost of operating firm j. Let the PDF of the distribution of f be v and the CDF be V. Then

$$\psi_0 = \int_0^{R_{j0}} v(f)df = V(R_{j0})$$

and

$$d\psi_0 = v(R_{j0})dR_{j0},$$

where $dR_{j0} = (c_{jk0}^{\mathbb{P}} + c_{jk0}^{\mathbb{R}})d\xi + \xi dc_{jk0}^{\mathbb{P}}$. The COVID shock thus affects the number of firms directly: COVID restrictions ($\xi < 1$) reduce the number of products that firms can sell and thus push some firms into the red forcing them to exit. There is also an indirect channel: the COVID shock increases the spending of the poor household on remaining products, which helps to mitigate firm exit. In the absence of steady-state transfers to poor households ($T^{\mathbb{P}} = 0$), the indirect channel is not operational and firm exit is entirely determined by the distribution of firms' fixed costs:

$$\left.\frac{d\psi_0}{d\xi}\right|_{T^{\mathbb{P}}=0} = v(R_{j0})\left(c_{jk0}^{\mathbb{P}} + c_{jk0}^{\mathbb{R}}\right). \tag{21}$$

As discussed in Section 3.2 (below Proposition 2), in the absence of transfers, restrictions decrease poor-household aggregate spending to a degree that perfectly balances the decrease in poor-household aggregate income. Given product symmetry, there is no adjustment within product categories.

If transfers $T^{\mathbb{P}}$ are positive in the steady state, the increase in per-product household spending induced by the restrictions can help mitigate firm exit in response to COVID restrictions. The poor household spreads their transfer wealth over the fewer remaining products, which helps to support firm revenues and mitigate firm exit.

If we interpret fixed operating costs as the rental cost of the existing capital stock, then f_{j0} is the price of capital and firm exit is associated with a reduction in demand for the existing capital stock. If prices f_{j0} are rigid, there will be excess supply of capital. However, if prices f_{j0} are flexible, then then they will adjust downward to mitigate the effect of falling revenues on firm profits. Given the inelastic supply of capital and the symmetry of firms, the capital market clears once the mass of operating firms is at its steady-state level ($\psi = 1$). We collect these results in the following proposition.

Proposition 5: If fixed operating costs are rigid, COVID restrictions induce firm exit. If fixed operating costs are flexible, there is no exit (and hence the restriction multiplier is bounded above by unity).

Proof: Appendix.

Clearly, if costs of operation can be adjusted in response to the COVID shock (e.g., set $v(R_{j0}) = 0$), firm exit can be avoided entirely and thus the adverse effects of the COVID shock minimized. However, there are plenty of reasons to expect that capital costs may not be flexible, at least in the short run. Asymmetric information between capital owners and the firms that rent the capital is among the reasons for rigid capital prices. If capital is imperfectly substitutable such that owners have pricing power, then capital owners may be reluctant to adjust if they cannot identify which firms can pay and which cannot. Indeed, recent empirical evidence documents a strong role for asymmetric information in preventing renegotiations between capital owners and firms even when such renegotiations would otherwise benefit both (Gilje et al., 2020).

The experience of the pandemic to date is consistent with rigid costs. The number of small business owners plummeted at the fastest rate on record between February and April 2020 (Farlie 2020). The adverse experiences of small businesses has led to a sharp fall in household wages and income, especially for households with low income (Canjer et al., 2020). Household evictions have also accelerated according to data from the Eviction Lab, indicative of rigid housing rental prices.[15]

Note that the disproportionate fall in income for poor households documented by Canjer et al. (2020) is consistent with the model. Rich households receive income from their ownership of the numeraire and from the NMC sector, whereas poor households receive income only from the NMC sector. Therefore, adverse shocks to the NMC sector disproportionately reduce the income of poor households.[16]

Inequality and Firm Exit. The effect of restrictions on firm exit is stronger the higher is the income share of the poor, unless the distribution of fixed costs is strongly decreasing at R_{j0} – that is, unless the elasticity of the density function $\frac{R_{j0}v'(R_{j0})}{v(R_{j0})}$ is less than -1:

$$\left.\frac{d^2\psi_0}{d\xi d\kappa}\right|_{T^{\mathbb{P}}=0} = \left(1 + \frac{R_{j0}v'(R_{j0})}{v(R_{j0})}\right) v(R_{j0}) \frac{dc_{jk0}^{\mathbb{P}}}{d\kappa}$$

Higher income of the poor is associated with higher spending on each product and hence a greater revenue loss for each product that is restricted. If the higher spending does not sufficiently reduce the number of firms on the margin of exit, then in the face of high poor-household-income-shares, restrictions pull down revenues more and induce more firm exit. Therefore, higher inequality can mitigate the adverse effect of restrictions on firm exit.

3.4. Fiscal Policy

Government transfers to households and/or firms can mitigate the adverse effects of the restrictions.

[15] We do not explicitly model housing. However, one could interpret "firms" in the model as workers who produce a range of goods/services. Firm exit would then be similar to exiting the workforce (e.g., due to homelessness).

[16] The endowment good that is owned and consumed exclusively by the rich could be interpreted as high-end property. Anecdotally, rich households have continued (or increased) their consumption of vacation properties during the pandemic even as they have cut back on spending on services.

3.4.1 Transfers to Households.

Consider first transfers to low-income households. One can show that the effect on real GDP is

$$\frac{dY_0}{dT^{\mathbb{P}}} = Y_0 v\left(R_{j0}\right)\xi \; \frac{dc_{jk0}^{P}}{dT^{\mathbb{P}}} + \frac{dq_{jk0}^{\mathbb{P}}}{dT^{\mathbb{P}}} > 0. \qquad (22)$$

Transfers to low-income households of sufficient size can in principle fully offset secondary economic effects of the COVID-related restrictions. Transfers stimulate output through two channels. First, they increase spending on existing firms. Second, they induce firm entry, and this entry causes additional private-sector spending on the products of the entering firms. This firm entry margin is consistent with recent empirical evidence of the effects of fiscal stimulus (Auerbach et al., 2020b) and more generally with the effect of aggregate demand shocks (Campbell and Lapham, 2002).

Inspection of equation (18) implies that the effects of demand shocks on poor-household spending and consumption are falling in inequality. This, along with equation (22), implies that the effect of transfers on GDP is falling in inequality: the smaller is the income share of low-income households, the less spending circulates back as income to low-income households (and hence the less they can spend).

Proposition 6 (Fiscal multipliers and inequality): Transfers induce firm entry, amplifying the fiscal multiplier. Furthermore, the fiscal transfer multiplier is falling in inequality (rising in the income share of the poor κ): $\frac{d^2Y_0}{dT^{\mathbb{P}}d\kappa} > 0$.

Proof: See Appendix.

Intuitively, lower inequality increases the multiplier because the larger is the income share of low-income households, the more spending circulates back as income to low-income households (and hence the more they can spend). This is a rather surprising result, given that inequality has often been associated with large shares of credit-constrained households (and hence potentially large fiscal multipliers, as in Brinca et al. 2016 and Lee 2020). However, recent empirical evidence documents an inverse relationship between fiscal effects and inequality (Miranda-Pinto et al. 2020b; Yang 2017). Our theory and this evidence implies that household-level transfers are less effective during the recent episode of rising inequality.

3.4.2 Transfers to Firms.

An alternative to household-level transfers is to provide transfers $T^{\mathbb{F}}$ to firms. If the government cannot target the firm transfers but instead must allocate across all firms, then firm-level transfers are unambiguously less effective than household-level transfers: for a firm-level transfer, low-income households (which drive spending multipliers) only end up with a share of the transfer κ. More formally,

$$\frac{dY_0}{dT^{\mathbb{F}:All}} = \kappa\frac{dY_0}{dT^{\mathbb{P}}} < \frac{dY_0}{dT^{\mathbb{P}}},$$

where $T^{\mathbb{F}:All}$ is untargeted transfers across firms. Such transfers stimulate spending and firm entry, but their effect is diminished because a share $1-\kappa$ of the transfer ends up with the rich households, who do not contribute to spending multipliers.

While untargeted firm-level transfers are the least effective form of stimulus, targeted firm-level transfers are potentially the most effective. In particular, transfers that are targeted to firms on the margin of exit (those for which fixed costs are a large share of their revenues, $R_{j0} \approx f_{j0}$) prevent firm exit, which as discussed above is the mechanism though with COVID restrictions cause large multiplier effects. In particular, for each dollar targeted to marginal firms, the government would create $d\psi_0 = \frac{1}{v(R_{j0})}$ firms. Equivalently, if the mass of marginal firms is $v(R_{j0})$, the government must spend that amount to keep them alive. So $\frac{d\psi_0}{dT^{\mathbb{F}:Target}} = \frac{1}{v(R_{j0})}$. If the government can target such firms, the extra multiplier from targeted transfers $T^{\mathbb{F}:Target}$ (relative to untargeted firm-level transfers $T^{\mathbb{F}:All}$) is (see the Appendix for derivations)

$$\frac{dY_0}{dT^{\mathbb{F}:Target}} - \frac{dY_0}{dT^{\mathbb{F}:All}} = \left(\kappa\frac{dY_0}{dT^{\mathbb{P}}} + \frac{\partial Y_0}{\partial\psi_0}\frac{1}{v(R_{j0})}\right) - \left(\kappa\frac{dY_0}{dT^{\mathbb{P}}}\right) = \frac{\partial Y_0}{\partial\psi_0}\frac{1}{v(R_{j0})}, \tag{23}$$

where

$$\left.\frac{\partial Y_0}{\partial\psi_0}\right|_{T^{\mathbb{F}}=0} = Y_0. \tag{24}$$

For example, if fixed costs f are uniformly distributed on $[0, U]$, the marginal targeted tax dollar creates Y_0/U additional units of GDP compared to the marginal untargeted tax dollar.

In the absence of large changes in the distribution of fixed costs as revenues change, lower inequality (higher κ) is associated with larger relative benefits of targeted transfers. Each firm is associated with higher GDP the larger is the income share (and hence spending) of poor households. Therefore, saving these marginal firms is associated with larger net output gains.

The relative benefit (in terms of GDP per dollar spent) of targeted transfers to firms versus transfers to low-income households firms depends on how many firms are kept afloat with each dollar spent:

$$\begin{aligned}\frac{dY_0}{dT^{\mathbb{F}:Target}} - \frac{dY_0}{dT^{\mathbb{P}}} &= \left(\kappa \frac{dY_0}{dT^{\mathbb{P}}} + \frac{\partial Y_0}{\partial \psi_0} \frac{1}{v\left(R_{j0}\right)}\right) - \left(\frac{dY_0}{dT^{\mathbb{P}}}\right) \\ &= \frac{\partial Y_0}{\partial \psi_0} \frac{1}{v\left(R_{j0}\right)} - (1-\kappa) \frac{dY_0}{dT^{\mathbb{P}}},\end{aligned} \tag{25}$$

In this sense, the benefits of targeted transfers to firms (relative to transfers to the poor) are proportional to the indirect costs of the COVID19 restrictions (i.e., endogenous firm exit). If there are large restriction multipliers $\left(1 + \frac{d\psi_0}{d\xi}\right)$ (based on the joint distribution of fixed capital costs and firm revenues), then the relative benefits of targeted transfers are large and these benefits could be even larger if there are costs of reentry. Furthermore, lower inequality (higher κ) is associated with larger relative benefits of targeted firm-level transfers because as discussed above Y_0 is increasing in κ. Poor households also receive a higher initial (direct) share of the firm-level transfer, although this effect is offset by the fact that the effect of household transfers on output is increasing in κ.

Proposition 7 (The optimal composition of transfers): Targeted transfers to firms can be the most cost-effective means of mitigating a restriction multiplier above unity. The relative benefit of targeted transfers depends on the joint distribution of firm revenues and capital operating costs. The relative benefit is higher the greater is the income share of the poor (lower inequality) as long as $\left(\frac{v'(R_{j0})}{v(R_{j0})}\right)$ is not too large.

Proof: Appendix.

In practice it may be difficult to identify and target marginal firms, although the model offers some guidance. The firms most at risk of exit are those with relatively low profitability and for which capital operating costs are the largest or most rigid. Because of low profit margins, small businesses are likely to be particularly prone to exit (consistent with the evidence in Fairlie 2020), therefore implying an important role of targeted transfers to firms. If the government attempts to target firms but can do so only imperfectly, the multiplier will be in the range $\left[\frac{dY_0}{dT^{\mathbb{F}:All}}, \frac{dY_0}{dT^{\mathbb{F}:Target}}\right]$, with a larger effect the more of the stimulus goes to marginal firms.

An alternative policy to firm-level transfers is government loans to firms. But firms still need to cover their future-period fixed costs. Firms for which the present value of revenues in both periods falls below the present value of fixed costs will not be helped by loans (specifically, ψ_0 and ψ_1 can fall below 1 even if the government offers loans). Loans are only effective for the firms that cannot cover their fixed costs in the initial period but nonetheless earn profits in present value. Chetty et al. (2020) document a limited impact of loans to small businesses on firm employment and suggest that liquidity injections are insufficient for restoring employment at small businesses. Their evidence, interpreted through the lens of our model, is consistent with a decline in the present value of revenues sufficient to push firms to exit.

4. Conclusion

The COVID crisis has both raised immediate policy questions and highlighted key structural relationships in the economy. Inequality has risen, rich households have cut back spending on services, and firms have been pushed to the brink of exit in the face of rigid capital operating costs. We develop a model capable of addressing the roles of inequality and other key features of the pandemic economy. Our results have general implications for the macroeconomic effects of inequality and fiscal policy, while also providing guidance on the relative merits of alternative fiscal policies in the face of COVID restrictions on economic activity.

Our framework implies that rising inequality will drag down GDP, as will any additional reallocation of spending by rich households away from service sectors in which low-income households work. In the absence of these developments, the strongest macroeconomic threat associated with COVID19 is firm exit resulting from restrictions on the exchange of services and rigid capital costs, a pattern clearly observed in the data. Our model suggests that the adverse effects may be offset by transfers to households and firms. Furthermore, we show that transfers to firms on the margin of exit are particularly effective in countering economic contraction.

Our framework indicates a number of metrics that will be useful to monitor as the COVID19 crisis evolves. In the absence of rising inequality or reductions in spending by high-income households, nominal GDP will rebound to a level at or potentially beyond what it would have been in the absence of COVID. Rising inequality or reductions in spending by high-income households can mitigate this boom or cause a prolonged slump. Fiscal stimulus will be especially useful in the event of a slump, although its effect per dollar spent is decreasing in inequality. The other important metric is the prices of firms' operating capital, especially for firms that have large fixed operating costs relative to revenues and for multiproduct firms. Downward adjustment of capital prices can mitigate large restriction multipliers.

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Appendix

The following relationships are referenced throughout the proofs. First, in equilibrium the bond price Q equals the discount factor β. This follows from the rich household's first-order-condition with respect to the bond and the fact that $\lambda_{\mathbb{R}t}$ equals the marginal utility of the numeraire. In particular, the first-order condition for either household is $Q\lambda_0^h = \beta\lambda_1^h$. Since $\lambda_{\mathbb{R}t} = 1$, it follows that $Q = \beta$. Furthermore, it follows that $\lambda_{\mathbb{P}0} = \lambda_{\mathbb{P}1}$.

Second, in the steady-state (in the absence of shocks) households smooth their expenditure: $c_{jk1}^h = c_{jk0}^h$. Plugging firms' prices (equation (5)) into the household's demand (equation (4)) implies that equilibrium steady-state quantities are $q_{jkt}^h = \frac{\theta^h}{2\gamma}$ and equilibrium expenditure is

$$c_{jkt}^h = \frac{(\theta^h)^2}{4\gamma\lambda_{ht}}. \tag{26}$$

Expenditure smoothing (in steady-state) follows from $\lambda_{h0} = \lambda_{h1}$.

Third, the responses to shocks of product-level poor-household consumption at time t=0, prices at t=1, and poor-household expenditure in either period all move in the same direction. This is because the response of each can be captured by the response of the budget multiplier:

$$dc_{jk1}^{\mathbb{P}} = dc_{jk0}^{\mathbb{P}} = \frac{(\theta^{\mathbb{P}})^2}{4\gamma\lambda_{\mathbb{P}0}^2} d\lambda_{\mathbb{P}0} \tag{27}$$

$$dq_{jk0}^{\mathbb{P}} = -\frac{1}{\gamma} p_{jk0} d\lambda_{\mathbb{P}0} \tag{28}$$

$$dp_{jk1}^{\mathbb{P}} = -\frac{\theta^{\mathbb{P}}}{2\lambda_{\mathbb{P}0}^2} d\lambda_{\mathbb{P}0} \tag{29}$$

The results follow from $dc_{jk0}^{\mathbb{P}} = p_{jk0}^{\mathbb{P}} dq_{jk0}^{\mathbb{P}}$ due to fixed initial-period prices, $q_{jk0}^{\mathbb{P}} = -\frac{1}{\gamma} p_{jk0} d\lambda_{\mathbb{P}0}$, and $dc_{jkt}^{\mathbb{P}} = d\left(\frac{\theta^2}{4\gamma\lambda_{\mathbb{P}1}}\right) = d\left(\frac{\theta^2}{4\gamma\lambda_{\mathbb{P}0}}\right) = -\frac{\theta^2}{4\gamma\lambda_{\mathbb{P}0}^2} d\lambda_{\mathbb{P}0}$. These relationships imply that we can infer the direction of output per product in the initial period and the direction of prices in the future from the direction of spending in either period (or equivalently, from the response of the budget multiplier).

Proof of Proposition 1: Total differentiation of (12), after imposing the steady-state values $\xi, \psi_0, \psi_1 = 1$ and $dc_{jk1}^{\mathbb{P}} = dc_{jk0}^{\mathbb{P}}$, yields

$$\frac{T^{\mathbb{P}}}{1+Q}d\xi+\frac{T^{\mathbb{P}}}{1+Q}d\psi_0+\left(c_{jk1}^{\mathbb{P}}(1-Q\kappa)-Qc_{jk0}^{\mathbb{R}}\kappa\right)d\psi_1+(1+Q)(1-\kappa)dc_{jk0}^{\mathbb{P}}$$
$$=(1+Q)\kappa dc_{jk0}^{\mathbb{R}}+dT^{\mathbb{P}}+(1+Q)\left[c_{jk0}^{\mathbb{P}}+c_{jk0}^{\mathbb{R}}\right]d\kappa. \tag{30}$$

It follows that around a steady-state in which $T^{\mathbb{P}}=0$ (and for simplicity setting $d\psi_1=0$):

$$\frac{dc_{jk0}^{\mathbb{P}}}{d\kappa}=\frac{c_{jk0}^{\mathbb{P}}+c_{jk0}^{\mathbb{R}}}{1-\kappa}\ \Rightarrow\ \frac{dq_{jk0}^{\mathbb{P}}}{d\kappa}=\frac{1}{\bar{p}_{jk0}^{\mathbb{P}}}\frac{c_{jk0}^{\mathbb{P}}+c_{jk0}^{\mathbb{R}}}{(1-\kappa)}$$
$$\frac{dc_{jk0}^{\mathbb{P}}}{dc_{jk0}^{\mathbb{R}}}=\frac{\kappa}{1-\kappa}\ \Rightarrow\ \frac{dq_{jk0}^{\mathbb{P}}}{dc_{jk0}^{\mathbb{R}}}=\frac{1}{\bar{p}_{jk0}^{\mathbb{P}}}\frac{\kappa}{1-\kappa} \tag{31}$$
$$\frac{dc_{jk0}^{\mathbb{P}}}{dT^{\mathbb{P}}}=\frac{1}{(1+Q)(1-\kappa)}\ \Rightarrow\ \frac{dq_{jk0}^{\mathbb{P}}}{dT^{\mathbb{P}}}=\frac{1}{\bar{p}_{jk0}^{\mathbb{P}}}\frac{1}{(1+Q)(1-\kappa)}.$$

■

Proof of Proposition 2: The total derivative of initial-period GDP (from (8)) is

$$dY_0=Y_0(d\xi+d\psi_0)+dq_{jk0}^{\mathbb{P}}+dq_{jk0}^{\mathbb{R}}. \tag{32}$$

It follows that (imposing $\left.\frac{dq_{jk0}^{\mathbb{P}}}{d\xi}\right|_{T^{\mathbb{P}}=0}=0$ from (30) and $\frac{dq_{jk0}^{\mathbb{R}}}{d\xi}=0$ from (6))

$$\left.\frac{dY_0}{d\xi}\right|_{T^{\mathbb{P}}=0}=Y_0\left(1+\frac{d\psi_0}{d\xi}\right).$$

■

Proof of Proposition 3: $dc_{jk0}^{\mathbb{P}}$ and $dp_{jk1}^{\mathbb{P}}$ are related by substituting $d\lambda_{\mathbb{P}0}$ out of (27) and (29):

$$dp_{jk1}^{\mathbb{P}}=\frac{2}{\theta^{\mathbb{P}}}\,dc_{jk0}^{\mathbb{P}}.$$

From (30), we can write

$$\frac{dc_{jk0}^{\mathbb{P}}}{d\xi}=-\frac{T^{\mathbb{P}}\left(1+\frac{d\psi_0}{d\xi}\right)}{(1+Q)^2(1-\kappa)}. \tag{33}$$

Therefore, $\frac{dp_{jk1}^{\mathbb{P}}}{d\xi}=-\frac{2}{\theta^{\mathbb{P}}}\frac{T^{\mathbb{P}}\left(1+\frac{d\psi_0}{d\xi}\right)}{(1+Q)^2(1-\kappa)}$. When ξ falls, future prices increase. ■

Proof of Proposition 4: Note that by (33) and $dc_{jk0}^{\mathbb{P}}=p_{jk0}^{\mathbb{P}}dq_{jk0}^{\mathbb{P}}$ we can write

$$\frac{dq_{jk0}^{\mathbb{P}}}{d\xi} = -\frac{1}{p_{jk0}^{\mathbb{P}}} \frac{T^{\mathbb{P}}\left(1 + \frac{d\psi_0}{d\xi}\right)}{(1+Q)^2(1-\kappa)}.$$

Taking the derivative with respect to κ yields

$$\frac{d^2 q_{jk0}^{\mathbb{P}}}{d\xi d\kappa} = -\frac{T^{\mathbb{P}}}{p_{jk0}^{\mathbb{P}}} \left[\frac{(1-\kappa)\frac{d^2\psi_0}{d\xi d\kappa} + \left(1 + \frac{d\psi_0}{d\xi}\right)}{(1+Q)^2(1-\kappa)^2} \right],$$

which is positive if $(1-\kappa)\frac{d^2\psi_0}{d\xi d\kappa} + \left(1+\frac{d\psi_0}{d\xi}\right) > 0$. First, we must derive a closed-form expression for

$$\frac{d\psi_0}{d\xi} = v(R_{j0}) \left[\left(c_{jk0}^{\mathbb{P}} + c_{jk0}^{\mathbb{R}}\right) + \frac{dc_{jk0}^{\mathbb{P}}}{d\xi} \right],$$

Substituting in $\frac{dc_{jk0}^{\mathbb{P}}}{d\xi} = -\frac{T^{\mathbb{P}}\left(1+\frac{d\psi_0}{d\xi}\right)}{(1+Q)^2(1-\kappa)}$ from equation (30) and rearranging yields

$$\frac{d\psi_0}{d\xi} = v(R_{j0}) \left[\left(c_{jk0}^{\mathbb{P}} + c_{jk0}^{\mathbb{R}}\right) - \frac{T^{\mathbb{P}}\left(1 + \frac{d\psi_0}{d\xi}\right)}{(1+Q)^2(1-\kappa)} \right]$$

$$\frac{d\psi_0}{d\xi} = \frac{v(R_{j0})}{\left[1 + \xi \frac{T^{\mathbb{P}}}{(1+Q)^2(1-\kappa)}\right]} \left[\left(c_{jk0}^{\mathbb{P}} + c_{jk0}^{\mathbb{R}}\right) - \frac{T^{\mathbb{P}}}{(1+Q)^2(1-\kappa)} \right]$$

Using the steady-state relationship $c_{jk0}^{\mathbb{P}}(1-\kappa) - c_{jk0}^{\mathbb{R}}\kappa = \frac{T^{\mathbb{P}}}{(1+Q)}$, this expression can be rewritten as

$$\frac{d\psi_0}{d\xi} = \frac{v(R_{j0})}{\left[1 + \xi \frac{T^{\mathbb{P}}}{(1+Q)^2(1-\kappa)}\right]} \left[\frac{T^{\mathbb{P}} Q}{(1+Q)^2(1-\kappa)} + \frac{c_{jk0}^{\mathbb{R}}}{(1-\kappa)} \right]. \quad (34)$$

which is strictly positive. The next step is to derive $\frac{d^2\psi_0}{d\xi d\kappa}$. Taking the derivative of (34) with respect to κ yields

$$\left(\frac{d^2\psi_0}{d\xi d\kappa}\right) = \frac{v(R_{j0})}{\left[1+\frac{T^{\mathbb{P}}}{(1+Q)^2(1-\kappa)}\right]}\left[\frac{T^{\mathbb{P}}Q}{(1+Q)^2(1-\kappa)^2}+\frac{c^{\mathbb{R}}_{jk0}}{(1-\kappa)^2}\right]$$
$$+\frac{\left[\frac{T^{\mathbb{P}}Q}{(1+Q)^2(1-\kappa)}+\frac{c^{\mathbb{R}}_{jk0}}{(1-\kappa)}\right]}{\left[1+\frac{T^{\mathbb{P}}}{(1+Q)^2(1-\kappa)}\right]^2}\left(\left(1+\frac{T^{\mathbb{P}}}{(1+Q)^2(1-\kappa)}\right)v'(R_{j0})\frac{dR_{j0}}{d\kappa}\right.$$
$$\left. - v(R_{j0})\frac{T^{\mathbb{P}}}{(1+Q)^2(1-\kappa)^2}\right).$$

It can be shown that this reduces to

$$\left(\frac{d^2\psi_0}{d\xi d\kappa}\right) = \frac{1}{1-\kappa}\frac{d\psi_0}{d\xi}\left\{1+\frac{1}{\left[1+\frac{T^{\mathbb{P}}}{(1+Q)^2(1-\kappa)}\right]}\left(\left(1+\frac{T^{\mathbb{P}}}{(1+Q)^2}\right)\frac{v'(R_{j0})}{v(R_{j0})}\frac{dR_{j0}}{d\kappa}-\frac{T^{\mathbb{P}}}{(1+Q)^2(1-\kappa)}\right)\right\}.$$

Since $\frac{dR_{j0}}{d\kappa} > 0$, this expression is strictly greater than $\Psi \equiv \frac{1}{1-\kappa}\frac{d\psi_0}{d\xi}\left\{1-\frac{A}{[1+A]}\right\}$, where $A \equiv \frac{T^{\mathbb{P}}}{(1+Q)^2(1-\kappa)}$.

Therefore it is sufficient to prove that $(1-\kappa)\Psi + \left(1+\frac{d\psi_0}{d\xi}\right) > 0$. Indeed, we have

$\frac{d\psi_0}{d\xi}\left\{1-\frac{A}{[1+A]}\right\}+\left(1+\frac{d\psi_0}{d\xi}\right) = \frac{d\psi_0}{d\xi}\left(2-\frac{A}{[1+A]}\right)+1 > 0.$ ■

Proof of Proposition 5: The effect of restrictions in the presence of rigid capital costs follows from (21). When capital costs are flexible, the price of capital will adjust to clear the capital market: Let r be the rate at which capital is rented out to firms, and let γ_j be the fixed capital requirement of firm j (so that $f_j = r\gamma_j$). If the capital market is flexible, then the rate will adjust so that the rental rate equals the revenues of the marginal firm. Without loss of generality, we can assume that the rental rate in the absence of COVID restrictions is such that there is a unit mass of firms: $\int_0^1 \gamma_j dj = K$. Covid restrictions shifts in the demand for capital (as firms' revenues fall). Given the inelastic supply of capital, a flexible capital market implies that r adjusts so that in equilibrium there remains a unit mass of firms, with the price determined by the firm on the margin of exit. ■

Proof of Proposition 6: Substitute for $\frac{dc^{P}_{jk0}}{dT^{\mathbb{P}}}$ and $\frac{dq^{\mathbb{P}}_{jk0}}{dT^{\mathbb{P}}}$ from (18) into the expression for $\frac{dY_0}{dT^{\mathbb{P}}}$ in (22):

$$\frac{dY_0}{dT^{\mathbb{P}}} = Y_0 v(R_{j0})\xi\ \frac{1}{(1+Q)(1-\kappa)}+\frac{1}{p^{\mathbb{P}}_{jk0}}\frac{1}{(1+Q)(1-\kappa)}$$

Taking the total derivative yields

$$\frac{d^2 Y_0}{dT^{\mathbb{P}}} = Y_0 v(R_{j0})\xi \ \frac{1}{(1+Q)(1-\kappa)}\left[dY_0 + v'(R_{j0})dR_{j0}\right] + \frac{1}{(1+Q)}\left(Y_0 v(R_{j0})\xi + \frac{1}{p_{jk0}^{\mathbb{P}}}\right)\frac{1}{(1-\kappa)^2}d\kappa$$

$\frac{d^2 Y_0}{dT^P d\kappa} > 0$ follows from $\frac{dR_{j0}}{d\kappa}, \frac{dY_0}{d\kappa} > 0$. ■

Proof of Proposition 7 (Effect of Targeted Firm-Level Transfers):

For each dollar targeted to marginal firms, the government would create $d\psi_0 = \frac{1}{v(R_{jt})}$ firms. Equivalently, if the mass of marginal firms is $v(R_{jt})$, the government must spend that amount to keep them alive. So $\frac{d\psi_0}{dT^{Target}} = \frac{1}{v(R_{jt})}$.

$\kappa dT^{\mathbb{F}:Target}$ would also be transferred to households (as they own a share κ of capital).

Therefore

$$\frac{dY_0}{dT^{\mathbb{F}:Target}} = \frac{\partial Y_0}{\partial T^{\mathbb{P}}}\frac{dT^{\mathbb{P}}}{dT^{\mathbb{F}:Target}} + \frac{\partial Y_0}{\partial \psi_0}\ \frac{d\psi_0}{dT^{\mathbb{F}:Target}}$$

$$\frac{dY_0}{dT^{\mathbb{F}:Target}} = \kappa\frac{dY_0}{dT^{\mathbb{P}}} + \frac{\partial Y_0}{\partial \psi_0}\ \frac{1}{v(R_{j0})}$$

$$\frac{dY_0}{dT^{\mathbb{F}:Target}} = \kappa\frac{dY_0}{dT^{\mathbb{P}}} + \frac{\partial Y_0}{\partial \psi_0}\ \frac{1}{v(R_{j0})}$$

If the government could not target firms – but rather spent across all firms, it would create only

$$\frac{dY_0}{dT^{\mathbb{F}:Firms}} = \frac{\partial Y_0}{\partial T^{\mathbb{P}}}\frac{dT^{\mathbb{P}}}{dT^{\mathbb{F}:Firms}} = \kappa\frac{dY_0}{dT^{\mathbb{P}}}.$$

Targeted firm transfers have an additional multiplier effect given by

$$\frac{dY_0}{dT^{\mathbb{F}:Target}} - \frac{dY_0}{dT^{\mathbb{F}:Firms}} = \frac{\partial Y_0}{\partial \psi_0}\ \frac{1}{v(R_{j0})},$$

where

$$\left.\frac{\partial Y_0}{\partial \psi_0}\right|_{T^{\mathbb{P}}=0} = Y_0.$$

The relationship between the net benefit of targeted transfers and inequality is

$$\frac{d}{d\kappa}\left(Y_0 \frac{1}{v(R_{j0})}\right) = \frac{1}{v(R_{j0})}\frac{dY_0}{d\kappa} - Y_0\frac{v(R_{j0})}{v^2(R_{j0})}\frac{dR_{j0}}{d\kappa} = \frac{1}{v(R_{j0})}\frac{1}{\bar{p}^{\mathbb{P}}_{jk0}}\frac{c^{\mathbb{P}}_{jk0} + c^{\mathbb{R}}_{jk0}}{(1-\kappa)} - Y_0\frac{v'(R_{j0})}{v^2(R_{j0})}\frac{dc^{\mathbb{P}}_{jk0}}{d\kappa}.$$

Substituting in the steady-state relationship $c^{\mathbb{P}}_{jk0}\big|_{T^{\mathbb{P}}=0}\frac{\kappa}{(1-\kappa)}c^{\mathbb{R}}_{jk0}$, as well as expressions for $\bar{p}^{\mathbb{P}}_{jk0}$ and $\frac{dc^{\mathbb{P}}_{jk0}}{d\kappa}$, this relationship can be written as (see Lemma 1 below for details)

$$\frac{d}{d\kappa}\left(Y_0 \frac{1}{v(R_{j0})}\right) = \frac{c^{\mathbb{R}}_{jk0}}{v(R_{j0})(1-\kappa)^2}\left[\frac{\kappa}{(1-\kappa)}\frac{(\theta^{\mathbb{R}})^2}{(\theta^{\mathbb{P}})^2} - \frac{(\theta^{\mathbb{P}}+\theta^{\mathbb{R}})}{2\gamma}\frac{v'(R_{j0})}{v(R_{j0})}\right].$$

As long as as long as the percent change of the distribution v'/v is not too high (specifically, as long as $\frac{v'}{v} < \frac{\kappa}{(1-\kappa)}\frac{(\theta^{\mathbb{R}})^2}{(\theta^{\mathbb{P}})^2}\frac{2\gamma}{\theta^{\mathbb{P}}+\theta^{\mathbb{R}}}$), a higher income share of the poor (lower inequality) is associated with a higher net benefit of targeted transfers. Note that the same result applies to the net benefit of target firm-level transfers relative to transfers to poor households,

$$\frac{dY_0}{dT^{\mathbb{F}:Target}} - \frac{dY_0}{dT^{\mathbb{P}}} = Y_0 \frac{1}{v(R_{j0})} - (1-\kappa)\frac{dY_0}{dT^{\mathbb{P}}},$$

since it is straightforward to show that $\frac{d}{d\kappa}\left(-(1-\kappa)\frac{dY_0}{dT^{\mathbb{P}}}\right) = 0.$■

Lemma 1:

$$\frac{d}{d\kappa}\left(Y_0 \frac{1}{v(R_{j0})}\right) = \frac{1}{v(R_{j0})}\frac{dY_0}{d\kappa} - Y_0\frac{v(R_{j0})}{v^2(R_{j0})}\frac{dR_{j0}}{d\kappa} = \frac{1}{v(R_{j0})}\frac{1}{\bar{p}^{\mathbb{P}}_{jk0}}\frac{c^{\mathbb{P}}_{jk0} + c^{\mathbb{R}}_{jk0}}{(1-\kappa)} - Y_0\frac{v'(R_{j0})}{v^2(R_{j0})}\frac{dc^{\mathbb{P}}_{jk0}}{d\kappa}$$

Sub in $c^{\mathbb{P}}_{jk0}\big|_{T^{\mathbb{P}}=0}\frac{\kappa}{(1-\kappa)}c^{\mathbb{R}}_{jk0}$ and $\frac{dc^{\mathbb{P}}_{jk0}}{d\kappa}\Big|_{T^{\mathbb{P}}=0} = \frac{c^{\mathbb{P}}_{jk0}+c^{\mathbb{R}}_{jk0}}{1-\kappa}$

$$\frac{d}{d\kappa}\left(Y_0 \frac{1}{v(R_{j0})}\right) = \frac{c^{\mathbb{R}}_{jk0}}{v(R_{j0})(1-\kappa)^2}\left[\frac{1}{\bar{p}^{\mathbb{P}}_{jk0}} - Y_0\frac{v'(R_{j0})}{v(R_{j0})}\right]$$

Substitute in $\bar{p}^{\mathbb{P}}_{jk0} = \frac{\theta^{\mathbb{P}}}{2\lambda_{\mathbb{P}}}$ and $Y_0 = \frac{\theta^{\mathbb{P}}}{2\gamma} + \frac{\theta^{R}}{2\gamma}$. Note we can solve for λ from $c^{\mathbb{P}}_{jk0}\big|_{T^{\mathbb{P}}=0}\frac{\kappa}{(1-\kappa)}c^{\mathbb{R}}_{jk0}$, where $c^{\mathbb{P}}_{jk0} = \frac{\theta^2}{4\gamma\lambda}$. Specifically, $\lambda_{\mathbb{P}} = \frac{\theta^2}{4\gamma c^{\mathbb{R}}_{jk0}}\frac{1-\kappa}{\kappa}$, where $c^{\mathbb{R}}_{jk0} = \frac{(\theta^{\mathbb{R}})^2}{4\gamma}$.

Then

$$\frac{d}{d\kappa}\left(Y_0\ \frac{1}{v\left(R_{j0}\right)}\right) = \frac{c_{jk0}^{\mathbb{R}}}{v\left(R_{j0}\right)(1-\kappa)^2}\left[\frac{\kappa}{(1-\kappa)}\frac{(\theta^{\mathbb{R}})^2}{(\theta^{\mathbb{P}})^2} - \frac{(\theta^{\mathbb{P}}+\theta^{\mathbb{R}})}{2\gamma}\frac{v'\left(R_{j0}\right)}{v\left(R_{j0}\right)}\right]$$

■