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MICRO-ESTIMATION OF THE DEMAND FOR SCHOOLING

Evidence from Michigan and Massachusetts

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Both micro and macro studies of the demand for local public schooling are quite common. Micro studies, which rely explicitly on the use of survey instruments, allow for more extensive data sets and more variables, but are subject to response biases. This paper uses data for the state of Massachusetts to replicate an earlier Michigan micro study of school spending. The authors find generally similar results in both states; in particular, they find an income elasticity of demand that is substantially less than one. They suggest that such a result is consistent with a more general model that includes an educational production function.

1. Introduction

The specification and estimation of the demand for local public schooling and other publicly provided goods has been a research topic for several decades. The continuing interest in the subject is due to the value that income and price elasticities can have for state and local public finance policy. For example, these elasticities can help in the design of grant programs and in the prediction of program effects. Similarly, they can be valuable in the evaluation and design of state and local tax structures.

Both micro and macro studies of the demand for local public schooling are now quite common. The more standard macro studies, which rely on precinct voting or school district expenditure data, frequently require 'median voter' assumptions. Sometimes data are limited; often there is an uncomfort-

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ably small set of exogenous variables. In particular, with macro studies it is virtually impossible to distinguish the effects of individual characteristics from neighborhood characteristics. Micro studies, which rely explicitly on the use of survey instruments, involve more extensive data sets and more variables. But they are subject to response biases and measurement errors that may be associated with the survey process.

In this paper we update our knowledge of the micro side of the picture, and at the same time confront a number of troubling issues that overlay the micro-versus-macro debate. We use two vehicles as the basis for this inquiry – the first is the 'Michigan' data base that has served us and our co-authors in previous micro demand function studies. The second is a comparable 'Massachusetts' data base that has been utilized by Ladd, Christopherson, and Reid.

One important issue that arises in the Michigan studies relates to the size of the income elasticity of demand for public schooling. Macro studies tend to generate substantially larger elasticities than do micro studies. A similar pattern arises when we replicate the micro approach using Massachusetts data. We discuss the general sources of these differences and emphasize one particular interpretation that is consistent with the data. A second, related issue, has to do with the nature of the education production function. To what extent are income and price elasticities of the demand for school outputs likely to be biased, when *spending*, rather than *output* demand functions are estimated?

We begin in section 2 with a general dicussion of the micro data and the robustness of the micro demand parameters. We do so by comparing demand functions estimated using the Michigan and Massachusetts data sets. The Massachusetts study replicates most of the earlier Michigan work. In particular, it supports the view that micro income elasticities are lower than their macro counterparts. We treat the question of why this is so in the theoretical part of section 3. In that section we distinguish spending from output demand functions, and ask whether the income elasticity of spending demand provides a biased estimate of the income elasticity of output. Section 4 concludes with some brief comments.

2. The Michigan and Massachusetts studies

2.1. The survey instrument

To what extent do the results of micro demand studies vary from place to place and from time to time. To what extent do the differences in survey questions affect the results? A partial answer to these questions comes from a direct comparison of the Michigan and Massachusetts studies.

The Michigan study is based on a subsample of 1,021 homeowners from a

random survey of 2,001 individuals taken immediately after the November, 1978 general election. 1978 was a year in which tax limitation was a hot issue, and there were two important tax limitation issues on the ballot at the time. During the telephone survey each respondent was asked:

> 'Do you think the state and local governments should be spending more, spending less, or about the same amount on the local public school system as they are spending now?'

If the response to this question was 'more', it was followed by a second question:

> 'If your taxes had to be raised to pay for the additional expenditures on local public schools would you still favor an increase in expenditure in this area?'

Individuals who responded 'yes' to this second question was recorded as favoring 'more' expenditure on schools. If the response was 'no', the respondent was recorded as desiring 'the same' level of expenditure. Of the sample of homeowners, approximately 25 percent appeared in the 'more' category, 58 percent were recorded as 'same', and only 17 percent claimed to want 'less'.

The Massachusetts survey, using a stratified random survey of 1,561 heads of households (767 of whom were homeowners in our analysis), was taken in 1980, following another tax limitation amendment election. The Massachusetts survey instrument contained two sets of questions relating to school spending preference. One qualitative question (involving five choices, rather than three) asked respondents views about public services generally. With respect to public elementary and high school education, however, a more quantitative approach was used. Respondents were asked:

> 'Compared to what the *state* government now spends, by what percentage, if any, would you like to see *state* government taxing and spending increase or decrease. You may answer any percent increase or decrease from 1% to 100%, or tell me you want it to stay the same... And, by what percentage, if any, would you like to see *local public school* taxes and spending increase or decrease?'

We have chosen to form a categorical variable from these quantitative responses, by attributing the response 'less' to anyone who stated that he would like to see local public school taxes and spending decreased, 'more' to anyone who would like to see both spending and taxes increased, and 'same' to all those who wanted to maintain the current spending level. The decision to collapse a continuous variable into a discrete one is a judgmental one: we felt that the possible loss of information (and therefore reduced efficiency) was outweighed by the fact that specific quantitative responses were likely to

contain substantial errors, in part because many respondents do not understand the concept of percentage. Of course, one benefit of the qualitative approach is that it allows us to evaluate the ordered probit estimation approach of Bergstrom, Rubinfeld and Shapiro (1982) using comparable data sets.[1]

Massachusetts voters were substantially less enthusiastic about an increased school budget than were their Michigan counterparts. Fifty-eight percent stated that they wanted less spending, 34 percent wanted the same, and only 13 percent desired more spending on education. This was undoubtedly due in part to differences in the strength of the tax limitation movements in the two states – in Massachusetts Proposition 2-1/2, a stringent tax limitation amendment, passed relatively easily. In Michigan (two years earlier) the stringent amendment (Tisch) was easily defeated, while a relatively mild amendment (Headlee) barely passed. But, the major difference in responses is most likely due to the differences in spending levels between the states. Michigan districts had a mean spending level of $1,708 per pupil, while the Massachusetts mean level was substantially higher, at $2,345.

While spending levels and qualitative responses differ substantially among the states, demand functions for school spending (and the underlying preferences for school outputs) may or may not be similar. We turn to the spending demand functions in the following section.

2.2. Demand estimation

The micro approach to demand estimation has been detailed elsewhere. Briefly, it works as follows. Let, the household's desired demand for school expenditures per pupil be given by

$$\ln(D) = \beta_0 + \beta_1 \ln(t) + \beta_2 \ln(Y) + \beta_3 X + \varepsilon, \tag{1}$$

where D is the respondent's desired level of per-student school expenditures in this school district, t is the tax price, the estimated tax cost to the individual of an additional dollar of expenditures per pupil in his district, Y is income after local taxes (and inclusive of exogenous grants per capita), and X is a vector of other individual and neighborhood characteristics.

Not all individuals within a jurisdiction get to consume the level of school expenditures that is desired. Therefore, we represent the difference between the logs of the provision of per-pupil spending (E) and desired spending by:

$$E - \ln(D) = \gamma_0 + \gamma_1 X + u = v, \tag{2}$$

[1]Most of the general results using the Massachusetts data set would be unchanged were the continuous form of the variable used in a demand function analysis.

where E is the logarithm of the jurisdiction's per-pupil spending on education, u is a random disturbance term, and v is defined as the difference between actual and desired expenditures.

The variable v is not directly observable , but information concerning v is available from the More-Same-Less questions described earlier. Specifically, we can describe these three qualitative ranges in terms of v and a threshold level δ, using the following three equations:

$$\text{More} \quad \text{if} \quad v<\delta, \tag{3}$$

$$\text{Same} \quad \text{if} \quad -\delta<v<\delta, \tag{4}$$

$$\text{Less} \quad \text{if} \quad \delta>v. \tag{5}$$

Substituting from (1) and (2), the conditions then become:

$$\text{More} \quad \text{if} \quad \varepsilon>E-\beta_0-\beta_1\log(t)-\beta_2\log(Y)-\beta_3X+\delta, \tag{6}$$

$$\text{Same} \quad \text{if} \quad E-\beta_0-\beta_1\log(t)-\beta_2\log(Y)-\beta_3X-\delta<\varepsilon$$

$$<E-\beta_0-\beta_1\log(t)-\beta_2\log(Y)-\beta_3X+\delta, \tag{7}$$

$$\text{Less} \quad \text{if} \quad \varepsilon<E-\beta_0-\beta_1\log(t)-\beta_2\log(Y)-\beta_3X-\delta. \tag{8}$$

Using eq. (6), (7), and (8), we can estimate all of the parameters of the demand equation using full-information maximum-likelihood estimation. This estimation procedure reflects the fact that individuals may sort themselves into jurisdictions on the basis of the level of educational spending. Thus, spending demands are a function of income, price and other variables from eq. (1). But, at the same time the school district in which a household lives and that district's political choice of spending level is a function of the migration decisions of households, as given by eq. (2).

To obtain consistent parameter estimates in eq. (1), the maximum-likelihood estimation procedure must account for the possibility of selection bias associated with eq. (2). The particular form of selection bias seen here is called 'Tiebout bias', in recognition of the importance of mobility and jurisdiction choice that has been given by the work of Charles Tiebout.

2.3. Comparing the Michigan and Massachusetts results

In making our comparisons between the micro demand equations, we begin with models that have not been estimated with Tiebout bias corrections. There are two reasons for this. First, these corrections inevitably involve an arbitrary choice of instruments, which may make it more difficult

to interpret the results. Second, differences in the data sets make it impossible to use the same set of instruments to make the corrections in the Michigan and Massachusetts studies.

The basic results are given in table 1. The Michigan parameter estimates are described as Models 1 and 2, while the Massachusetts estimates are given as Models A, B, and C. The absolute value of the t-statistics appear in parentheses below the estimated coefficients. Table 2 contains the estimated price and income elasticities of demand associated with each of the five models.[2]

As can be seen, Model 1 and Model A are directly comparable. Model 2 is closely related, although not identical to Models B and C. (Unfortunately, not all of the survey questions used in Model 2 were included in the Massachusetts survey.) Model C is identical to Model B, except that Model C excludes all variables that were insignificantly different from zero in Model B.

While there are some important differences between the two sets of results, what is most striking to us is their similarity. Consider first the income and price elasticities. In the parsimonious models, the Massachusetts elasticities are slightly higher than the Michigan elasticities. In both cases, the income elasticities are relatively high, although less than one. A greater difference arises when we control for other demand-determining variables. The price elasticities do not change much, but the income elasticities fall using both data sets. Interestingly, the Michigan income elasticity is substantially lower than its Massachusetts counterpart.[3]

The substantial decline in the income elasticity when we move from macro to micro studies is fundamental. In our view, it reflects the fact that the true income elasticity, which measures by how much educational spending demand will increase in response to an increase in discretionary income, is relatively low. The higher elasticity associated with Models 1 and A is likely to be consistent with the income elasticities that are obtained from macro models that (because of data limitations) cannot accurately separate out individual from community characteristics. A failure to control for the effect of all individual characteristics, some of which may be correlated with variables that enter the educational production function, is likely to bias the estimated elasticities.

A comparison of the other estimated parameters of the Michigan and Massachusetts demand models show how closely similar they are. All but

[2]The Michigan estimates are taken from Bergstrom, Rubinfeld, and Shapiro (1982, table I, p. 1192).

[3]Note that this difference is due largely to the fact that the 'Log expenditure' coefficient increases substantially when we move from Model 1 to Model 2, but not when we move from Model A to Model B. Unfortunately, this important expenditure coefficient cannot be measured with great accuracy, because it depends only on interjurisdictional (and not individual) variation. As a result, it is possible that some of the change in coefficients is due to the differences in sampling procedures used in the two studies.

Table 1

Comparison of demand parameter estimates (with t-ratios).

Independent variable	Model 1	Model A	Model 2	Model B	Model C
Log expenditure	−0.261 (1.21)	−0.328 (1.75)	−0.430 (1.31)	−0.333 (1.65)	−0.387 (2.06)
Log income	0.217 (4.17)	0.305 (3.74)	0.164 (2.45)	0.238 (2.72)	0.269 (3.25)
Log tax price	−0.150 (3.12)	−0.236 (2.83)	−0.187 (3.46)	−0.234 (2.61)	−0.247 (2.91)
Black	–	–	1.130 (5.59)	0.628 (2.06)	–
Jewish	–	–	0.787 (2.48)	0.161 (0.91)	–
Catholic	–	–	−0.014 (0.17)	–	–
No. of kids age 1–5	–	–	0.244 (3.64)	0.157 (2.60)	0.149 (2.52)
No. of kids age 6–11	–	–	0.144 (2.44)	–	–
No. of kids age 12–16	–	–	0.029 (0.54)	–	–
No. of kids age 6–16	–	–	–	0.093 (2.48)	0.092 (2.53)
Child in non-public school	–	–	−0.232 (1.60)	–	–
Not high school grad.	–	–	−0.077 (0.75)	–	–
College grad.	–	–	0.175 (12.50)	0.137 (1.38)	–
Log enrollment	–	–	−0.162 (2.75)	−0.028 (1.51)	–
Log pupils per school	–	–	0.400 (2.68)	–	–
% black in district	–	–	0.006 (2.00)	–	–
% non-white in community	–	–	–	0.004 (0.28)	–
Republican	–	–	−0.043 (0.51)	–	–
Female	–	–	0.100 (1.32)	–	–
School employee	–	–	0.329 (2.35)	0.131 (0.81)	–
Age 65 or over	–	–	0.272 (1.86)	–	–
Retired or disabled	–	–	−0.354 (2.70)	–	–

Table 1 (*continued*)

Independent variable	Model 1	Model A	Model 2	Model B	Model C
Unemployed	–	–	−0.518 (2.06)	–	–
On welfare	–	–	0.184 (0.75)	–	–
Detroit	–	–	−0.351 (1.20)	–	–
Lower income, expect higher	–	–	−0.078 (0.45)	−0.102 (0.86)	–
Lower income, expect lower	–	–	−0.244 (2.16)	−0.018 (0.13)	–
Higher income, expect lower	–	–	−0.155 (0.89)	0.013 (0.06)	–
Higher income, expect higher	–	–	0.183 (1.95)	0.187 (1.62)	–
Log county avg. teacher salary	–	–	1.488 (2.14)	–	–
Log county avg. wage rate	–	–	1.353 (1.31)	–	–
Log median county income	–	–	−1.234 (1.46)	–	–
Log per capita city income	–	–	−0.067 (0.32)	–	–
No. of observations (N)	949	767	943	767	767

Table 2

Comparison of elasticities between the models.

	Model 1	Model A	Model 2	Model B	Model C
Income elasticity	0.83	0.931	0.38	0.715	0.695
Tax price elasticity	−0.57	−0.719	−0.43	−0.704	−0.638

one of the comparable variables have the same sign. (The exception is one of the four 'expectation' variables, which is statistically insignificant in both equations.) Indeed, most of the variables that are significant in one equation

Table 3

	Michigan			Massachusetts		
	Less	Same	More	Less	Same	More
Total	171(17%)	597(58%)	253(25%)	404(53%)	261(34%)	102(13%)
Education						
College grad	24(15%)	86(55%)	47(30%)	114(47%)	93(38%)	37(15%)
Not college grad	147(17%)	511(59%)	206(24%)	290(55%)	168(34%)	65(11%)
Race						
Black	2(3%)	29(39%)	44(58%)	5(42%)	3(25%)	4(33%)
Not black	169(18%)	568(60%)	209(22%)	399(53%)	258(34%)	98(13%)
Religion						
Jewish	0	9(64%)	5(36%)	22(44%)	19(38%)	9(18%)
Not Jewish	171(17%)	588(58%)	248(25%)	382(53%)	242(34%)	93(13%)

are also significant in the other. Nonetheless, some brief comments concerning the differences in the results are appropriate.[4]

In both Michigan and Massachusetts the demand for education is unaffected by the racial and ethnic make-up of the jurisdiction. (The coefficients of the percent black and percent non-white variables are both very close to zero.) However, in both states individual black voters demand substantially more spending on education than do their white counterparts, a result for which we do not have a convincing sociological or economic explanation.

Some comparisons of the respondents in the two surveys are given in table 3. In Michigan, approximately 7% of homeowners respondents were black. 58% of them wanted more spending on education (in comparison to 22% of the non-blacks), despite the fact that their average level of educational attainment was lower than their non-black counterparts. (8% of blacks and 16% of non-blacks were college educated.) In Massachusetts, with only 2% black respondents, the pattern was similar, but not as strong. (16% of blacks and 32% of non-blacks were college educated.) One-third wanted more spending, in comparison to only 13% of the non-blacks.

We do find a striking difference among Jewish voters; in Michigan demands are substantially higher than average, other things equal, while in Massachusetts there are no large differences. But Jewish voters were only 1% of respondent households in the Michigan sample (all of whom wanted the same or more spending), so that the result is not likely to be substantively important. In Massachusetts, with a 7% respondent Jewish sample (two-thirds of whom were college educated), 18% wanted more spending, as

[4]We performed a likelihood-ratio test of the null-hypothesis that the set of coefficients in the two ordered probits are identical. We rejected the null-hypothesis at the 5% level. This rejection should not be surprising in light of the relatively large sample size and the fact that a few coefficients differ substantially between probits.

Table 4

Income and desired per-pupil spending.

	Michigan		Massachusetts	
	Income	Spending	Income	Spending
Respond 'less'	$18,414	$1,708	$26,315	$2,345
Respond 'same'	20,233	1,711	28,347	2,382
Respond 'more'	23,048	1,694	31,859	2,230

opposed to 13% of the remaining respondents (less than one-third of whom were college educated).

Now, let us return to an important topic – the income elasticity of demand. The micro approach can yield a high elasticity if either or both of the following are true: (i) the mean level of per-pupil spending increases as the mean level of income of households increases across jurisdictions; (ii) for any given community, households with higher incomes are more likely to respond 'more' to our survey than are households with lower incomes. The former source of income elasticity is the focal point of the macro approach, while the latter can only be sorted out using the micro approach.

Table 4 shows these breakdowns for the Michigan and Massachusetts data sets. We can see that in both Michigan and Massachusetts there is a positive correlation between income and the response of 'more', 'same', and 'less'. Likewise, there is a small, positive correlation between income and per-pupil spending across jurisdictions. Thus, it is not surprising that macro studies generate relatively low income elasticities in both states. And, there is essentially no correlation between the 'more-same-less' response and the spending level of the jurisdiction in which the respondents live. This leaves the possibility that micro income elasticities could be *larger* than their macro counterparts. But, the results of table 1 tell us that once other variables are controlled for, the micro and macro results are not very different.

In Rubinfeld, Shapiro and Roberts (1987) we discussed the difficulties with the ordered probit analysis when educational quality affects respondents' residential choices. In this case the resulting estimates of the income and price elasticity will be biased, although the direction of the bias is not given ex ante by the theory. For the Michigan sample we found some evidence of an upward bias.

Roughly speaking, our technique for correcting for selectivity-induced bias is to include in the demand equation the residual from a reduced-form regression of expenditures on all demand variables and several instrumental variables. In the Michigan study we used as instruments dummy variables that distinguished school districts in large metropolitan areas and in Detroit and the last year's percentage change in expenditures. When these were included, the estimated income and price elasticities were 0.10 and −0.11, respectively – smaller than the uncorrected estimates. The implication was

that community choice induced an upward bias in the ordered probit estimates of income and price elasticities.

Because of the inability to get exactly the same instruments for Massachusetts, we were not able to duplicate the same selectivity bias experiment. In the most directly comparable correction, we used a city dummy and the change in total jurisdiction public spending as instruments. In this case, we found virtually no change in the parameters of the Massachusetts demand equation. In fact we found that we could reject the hypothesis that community sorting matters. But, in both Massachusetts and Michigan, Tiebout bias results are quite sensitive to the choice of instruments, leaving open the possibility that further research may generate different outcomes.

The Massachusetts analysis replicates the general micro approach to demand estimation that was applied to the earlier Michigan data set. It also replicates the fact that micro income elasticities are low in absolute terms, and low relative to macro elasticities. In the next section of the paper we consider some theoretical reasons why this might be so.

3. Why is the micro income elasticity of demand low?

Both micro and macro estimates of the income elasticity of demand for public education have been controversial. Hamilton, for example, applied the production model of Bradford, Malt and Oates (1969) to demonstrate that they are implausibly low. Hamilton's skepticism is based on the belief that estimates of educational demand parameters are biased because they use educational expenditures as the measure of educational output.

More specifically, it has been claimed that the income elasticity of demand for expenditures is smaller than that the income elasticity of demand for educational output (hereinafter, education) because school district income (as a proxy for omitted district household characteristics) is positively correlated with the productivity of expenditures in producing education.[5] Thus, while the demand for education is income elastic, it appears to be inelastic in micro studies because the richer a household, the less might be spent to achieve a given level of educational output.[6]

One direct reponse is that school expenditures is an input into the production of education that is worthy of independent study. A second response is that output is difficult to measure, and cannot therefore serve adequately as the basis for an empirical study. In fact, there is a considerably

[5]While there appears to be a significant relationship between performance on standardized tests and income, income generally explains very little of the variance in performance. See Summers and Wolfe (1977) for details.

[6]Gramlich and Rubinfeld (1982) provide some implicit support for this view. They suggest that positive macro elasticities may arise because public services are distributed in a prorich manner. Their study uses expenditure and not output data, but their conclusion is consistent with the view that within-school district outputs vary positively with family income.

large set of output variables to choose from. Consequently any empirical study of education would need to be multidimensional.

We concur with the view that omitted variables *may* cause macro income elasticities of education to be biased downward. But, we are doubtful as to whether the same bias applied to *micro* elasticities, because micro demand functions are better specified than are macro functions. For example, our micro demand functions distinguish between micro household income (which is used to measure the income elasticity) and mean district income (which serves as the proxy for omitted household characteristics).

In the remainder of this section we concentrate on three related questions that help to explain why low micro income elasticities are plausible: (1) whether an income elasticity of education obtained from a micro (or macro) model that does not contain an explicit educational production function provides a biased estimate of the true income elasticity of education; (2) whether a correctly estimated micro income elasticity of *expenditure* provides a biased measure of the correctly estimated income elasticity of *education*; and, (3) whether low income elasticities of education are reasonable with an appropriately specified model.[7] In doing so, we specify an explicit production function for education, rather than using district income as a proxy for omitted household characteristics. We do not rely, therefore, on the assumption that income itself is positively related to the marginal productivity of educational expenditures.[8]

Consider the optimization problem of a household (with children) that has preferences over a private good X and education E. Suppose, consistent with the household production model of Becker and Tomes (1979), that parents desire education for their children as a means of giving them future income-producing ability. Specifically, assume that Z is the income (per child) produced by educational expenditures and a vector, A, of all other inputs used in the production of childrens' utility. Parents can give private lessons, their own parental time or can provide material inheritances. While the output of education is richer than just income (e.g., appreciation of literature and art) we concentrate our analysis on children's income. (A different framework would be needed if we were to use test scores as the output measure.)

The concave production function that links the output Z to the inputs A and E is given by

$$Z = F(E, A). \tag{9}$$

[7]We note in passing that estimates of the income elasticity of expenditures may be biased upwards if expenditures and other (omitted) production inputs are complements, or biased downwards if expenditures and other inputs are substitutes.

[8]Roberts and Shapiro (1986) found little evidence to support this view.

The demand function for educational expenditures is derived from the solution to the following optimization problem:

$$\begin{aligned} &\max U(X, F(E, A)) \\ &\text{s.t.} \quad I - X - tE - NA = 0. \end{aligned} \tag{10}$$

$U(\cdot)$ is concave, I is household income (net of federal and state taxes), t is the household's tax share (of taxes that pay for per-pupil educational expenditures),[9] and N is the number of children in the household.

Now consider our first two questions. How does the income elasticity of output that we obtain from this more complex model compare to one in which E replaces Z in the utility function? Within the complex model how does the income elasticity of expenditures relate to the income elasticity of education? We can get some answers by redefining the questions slightly. In what way, if at all, does the omission of A in the production function of eq. (9) affect the income elasticity of education? How do the income elasticities of expenditures and output relate?[10]

To answer these questions we can use comparative statics to find (the details appear in Appendix B):

$$\begin{aligned} \frac{\mathrm{d}E}{\mathrm{d}I} &= [m_X F_E (tF_{AA} - NF_{EA})]/D, \\ \frac{\mathrm{d}A}{\mathrm{d}I} &= m_X F_E (NF_{EE} - tF_{AE})/D. \end{aligned} \tag{11}$$

The expression within the brackets is negative because both the utility and production functions are concave (subscripts denote partial derivatives). The determinant, D, of the Hessian formed by taking derivatives of the first-order conditions is negative since U is concave in X, E, and A and the Hessian is negative definite. Therefore, both $\mathrm{d}E/\mathrm{d}I$ and $\mathrm{d}A/\mathrm{d}I$ are positive. Excluding A from the production specification excludes variables that are positively

[9]Since most states finance public education with an ad valorem property tax and total revenues received must equal expenditures, the balanced budget condition requires that: $t = [AV_i/AV] \cdot P$, where AV_i is the assessed value of household i's property, AV is the total community assessed value and P is the student population in the school district.

[10]Hamilton deals with the omitted variable problem by treating a special case – including I in $F(\cdot)$ as a proxy variable. But, it seems preferable to include the variable A itself rather than its proxy, thus allowing childrearing decisions to be explicitly endogenous.

correlated with I.[11] Clearly, bias can arise because the simpler formulation does not account for the productivity of A or for the effects of A on the productivity of E in the production of Z.

In general,

$$\frac{\mathrm{d}Z}{\mathrm{d}I}=F_E(\mathrm{d}E/\mathrm{d}I)+F_A(\mathrm{d}A/\mathrm{d}I). \tag{12}$$

Since $\mathrm{d}A/\mathrm{d}I$ and F_A are both positive, the income elasticity of education that we obtain from the fully specified model is greater than the elasticity associated with the simpler model (in which E replaces Z), just as Hamilton suggested. Note, however, that the magnitude of $\mathrm{d}Z/\mathrm{d}I$ may be greater than, equal to, or less than the magnitude of $\mathrm{d}E/\mathrm{d}I$. Therefore, the income elasticity of expenditures may be biased either upward or downward in relation to the income elasticity of education.

To treat these issues further, and to get a better sense of the relationship between the income elasticities of expenditures and output we consider two interesting special cases. Consider first what happens when the production function is semi-linear: $Z=F(E)+A$ and A is the material inheritance that the children receive. In this case $F_{EA}=F_{AA}=0$, and it follows that $\mathrm{d}E/\mathrm{d}I=0$. Thus, when income from education and income from material goods are perfect substitutes (as implied by the semi-linear form), the income elasticity of demand for educational expenditures is zero. But, $\mathrm{d}A/\mathrm{d}I\geqq 0$, and therefore, the demand for education may be substantially responsive to income. In fact in this particular case the income elasticity of demand for education is

$$e_Z=\frac{\mathrm{d}A}{\mathrm{d}I}\frac{I}{Z}.$$

Now, substituting for $\mathrm{d}A/\mathrm{d}I$, using eq. (11) and the expression for D from Appendix B, we obtain the output elasticity

$$e_Z=\left[\frac{m_X}{Nm_X-m_F F_A}\right]\frac{I}{Z}.$$

Because $m_F<0$ and $F_A>0$, the term in the square bracket is less than one. The income elasticity of demand for output Z depends on this ratio, and on the size of the ratio I/Z. If parents want the children to be as least as well-off as they themselves are, then $I/Z\leqq 1$ and the demand for output Z is income

[11]Tomes (1981) has estimated the demand for material inheritances, but these are only one element in the production process. For consistency with this model, we need measures of educational trips, parents time with children, and other expenditures that affect the welfare of a child.

inelastic. The demand will only be income elastic for parents who want their children to be worse-off. Thus, in the example, the income elasticity of output is greater than the expenditure elasticity, but both are likely to be less than one.

Now consider a second example of a Cobb–Douglas production function in which the production function is separable in the logs of the inputs:

$$Z = E^{\alpha} A^{\beta}.$$

It is easy to show [this follows from (9)] that the income elasticity of demand for Z is the weighted sum of the income elasticities of demand for E and A, where the weights are 'α' and 'β', respectively. Specifically,

$$e_Z = \alpha e_E + \beta e_A,$$

where e is the income elasticity of demand for the subscripted variable. It is interesting to note that with a Cobb–Douglas or any homothetic production function the income elasticities of demand for E and A are equal.[12] In this case the income elasticity of demand for education is

$$e_Z = (\alpha + \beta) e_E.$$

If F is a constant returns to scale production, $\alpha + \beta = 1$, and the income elasticities of expenditures and output are equal. However, the expenditure elasticity will be greater than or less than output elasticity depending on whether there are decreasing or increasing returns.

This theoretical exercise makes empirical work very important. Clearly the income elasticity of demand for educational expenditures can understate or overstate the income elasticity of demand for education. Which is the case can only be answered by a satisfactory empirical study. In addition, an accurate estimate of the income elasticity of education requires a proper empirical specification, including an educational production function. Finally, low income elasticities of expenditures and education are *both* theoretically possible; whether and to what extent they arise is an empirical question.

4. Concluding remarks

We have seen that the micro-based demand approach applied to both the Massachusetts and Michigan data sets (despite the many differences between the data sets) generates broadly similar results. In particular, the income

[12]We thank David Wildasin for pointing out that if the production function is a homothetic branch of the preference function, the income elasticities of demand of all production function inputs are equal.

elasticity of demand for educational expenditures is substantially less than one in both states.

An important question for us is whether these estimates of income elasticity are biased towards zero. We have shown (as Hamilton suggests) that the income elasticity of expenditures as an input to the production of educational output may generate biased estimates of the income elasticity of output demand. However, as a theoretical matter, this bias could go in either direction – if biased at all, our estimates are as likely to be too high as too low. Finally, as an empirical matter we believe that the micro elasticities are less subject to this type of bias than are macro studies, simply because micro studies are better able to measure and therefore to control for the effects of potentially omitted variables.

Appendix A: Definitions of variables

1. In adapting the Massachusetts data base for our purposes we dropped observations when there were missing values for the dependent variable (constructed from question 5 of the Massachusetts survey), income, expenditures, or tax-price. We also dropped observations for towns 3, 8, 16, and 58, all of which have a very small number of pupils and an unusually high level of expenditures per-pupil.

2. The income variable is the respondent's estimated household income less property tax payments. The tax price variable is the ratio of the household's tax burden to the community's average household tax burden, multiplied by the number of pupils per household in the community. Expenditures per-pupil is given by total expenditures from all funds, minus regional school assessments paid by municipalities, minus regional school assessments paid by school committees, minus tuition from individuals, minus tuition from other Massachusetts districts, minus tuition from out-of-state districts.

3. The 'expectation' variables are defined in the following manner: 'Lower income, expect higher' is a dummy variable that is equal to 1 if the household is financially worse off now than two years ago and expected to be better off two years from now; and 0 otherwise. Other variables are defined similarly.

Appendix B: Derivation of comparative statics results

Individual decision:

$$\max U(X, F(E, A))$$

$$\text{s.t.} \quad I - X - tE - NA = 0.$$

Let the marginal rate of substitution, m, be given by

$$\frac{U_F}{U_X}=m(X,F(E,A)).$$

The first-order conditions are

$$mF_E=t,$$

$$NF_E-tF_A=0,$$

$$I-X-tE-NA=0.$$

By the concavity of the utility and the production functions, $m_X>0$, $m_F<0$, $F_{EE}\leqq 0$ and $F_{AA}\leqq 0$. We assume $F_E>0$, $F_A>0$ and $F_{EA}\gtreqq 0$. If N and t are constant, the first-order conditions imply that

$$m_X F_E\,\mathrm{d}X+(mF_E^2+mF_{EE})\,\mathrm{d}E+(m_F F_A F_E+mF_{EA})\,\mathrm{d}A=0,$$

$$(NF_{EE}-tF_{AE})\,\mathrm{d}E+(NF_{EA}-tF_{AA})\,\mathrm{d}A=0,$$

$$\mathrm{d}X+t\,\mathrm{d}E+N\,\mathrm{d}A=\mathrm{d}I.$$

Let

$$D=\begin{vmatrix} m_X F_E & (m_F F_E^2-mF_{EE}) & (m_F F_A F_E+mF_{EA}) \\ 0 & (NF_{EE}-tF_{AE}) & (NF_{EA}-tF_{AA}) \\ 1 & t & N \end{vmatrix},$$

$$\begin{aligned} D=m_X F_E[N(NF_{EE}-tF_{AE})-t(NF_{EA}-tF_{AA})] \\ +(m_F F_E^2+mF_{EE})(NF_{EA}-tF_{AA}) \\ -(m_F F_A F_E+mF_{AE})(NF_{EE}-tF_{AE})<0, \end{aligned}$$

$$D_{EI}=\begin{vmatrix} m_X F_E & 0 & (m_F F_A F_E+mF_{EA}) \\ 0 & 0 & (NF_{EA}-tF_{AA}) \\ 1 & 1 & N \end{vmatrix},$$

$$D_{AI}=\begin{vmatrix} m_X F_E & (m_F F_E^2+mF_{EE}) & 0 \\ 0 & (NF_{EE}-tF_{AE}) & 0 \\ 1 & t & 1 \end{vmatrix}.$$

Applying Cramer's rule

$$\frac{\mathrm{d}E}{\mathrm{d}I}=\frac{D_{EI}}{D}=\frac{m_X F_E(tF_{AA}-NF_{EA})}{D},$$

$$\frac{\mathrm{d}A}{\mathrm{d}I}=\frac{D_{AI}}{D}=\frac{m_X F_E(NF_{EE}-tF_{AE})}{D}.$$

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