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Chapter 30

QUANTITATIVE METHODS IN ANTITRUST

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This chapter surveys empirical methods that are used to evaluate antitrust issues relating to merger analysis, liability, impact, and damages. The chapter first describes a number of applications for traditional statistical methods that rely on reduced form estimation using cross-section or time-series data. It then examines the application of methods that uncover the structure of demand, including (1) the estimation of demand elasticities using market transactions data; (2) the use of transactions or bidding data, perhaps merged with information on buyer characteristics, to learn about the structure of preferences and to make inferences about the extent of buyer substitution between alternatives; and (3) the use of survey techniques. The chapter then raises a number of important conceptual issues that are relevant with respect to questions of liability, impact, and damages.

1. Introduction

Quantitative methods have been described and debated in detail elsewhere by a host of commentators.[1] This chapter provides a brief, critical overview of methodologies that have been used and the range of questions that they address.

Econometrics involves the application of statistics to the quantitative, or empirical measurement of relationships that flow from economic analysis. Multiple regression and other econometric methods have been used frequently in cases brought by the competition authorities and in private litigation. The issues raised include liability, damages, and class certification. Empirical methods can help courts to identify what happened and why. Frequently, this can be accomplished by using a multiple regression analysis that distinguishes among a number of competing factors that Are correlated with (i.e., related to) a fact pattern. The technique allows one to isolate a key relationship or critical influence from other competing explanations. In using multiple regression, it is important to distinguish correlation from causality. Evidence that two variables are correlated does not in itself support the conclusion that the two are causally linked. A central role for econometrics and other quantitative methods is to provide the underlying theoretical support for key propositions involving such issues as relevant

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1. *See, e.g.*, Franklin Fisher, *Multiple Regression in Legal Proceedings*, 80 COLUM. L. REV. 702 (1980); Daniel L. Rubinfeld & Peter O. Steiner, *Quantitative Methods in Antitrust Litigation*, 46 L. & CONTEMP. PROBS. 69 (1983); Daniel L. Rubinfeld, *Econometrics in the Courtroom*, 85 COLUM. L. REV. 1048 (1985) [hereinafter *Courtroom*]; Jonathan B. Baker, *Contemporary Empirical Merger Analysis*, 5 GEO. MASON L. REV. 347 (1997); Jerry A. Hausman & Gregory K. Leonard, *Economic Analysis of Differentiated Products Mergers Using Real World Data*, 5 GEO. MASON L. REV. 321 (1997); Jonathan B. Baker & Daniel L. Rubinfeld, *Empirical Methods in Antitrust Litigation: Review and Critique*, 1 AM. L. & ECON. REV. 386 (1999); ABA SECTION OF ANTITRUST LAW, ECONOMETRICS (John Harkrider & Daniel L. Rubinfeld eds., 2005).

product market in the analysis of mergers and monopolization, the likely price effects of mergers, the anticompetitive effects of a range of anticompetitive practices involving restraints of trade, and, of course, the damages that flow from wrongful activity.[2]

Because the range of applications of these empirical techniques is so broad, this review must necessarily be limited. The chapter begins by focusing on methodology, explaining why it is important in antitrust analysis to distinguish between reduced-form methods and methods that identify the structure of demand. Three approaches are considered: (1) estimating demand elasticities from data on market transactions, (2) using transactions or bidding data to learn about the structure of preferences and to make inferences about the extent of buyer substitution between alternatives, and (3) using survey techniques to evaluate demand structure.

Next, in the applications section, the chapter discusses some of the important methodological issues that arise when one is considering, first liability, and then antitrust injury and damages. The chapter first describes a number of applications for traditional statistical methods that rely on reduced form estimation using cross-section or time-series data. It then examines the various methods that can be used to uncover the structure of demand. The chapter then raises a number of important conceptual issues that are relevant with respect to questions of liability,[3] and then antitrust injury[4] and damages.[5]

2. Methodology

2.1. Reduced-form methods

The most common statistical method employed in antitrust litigation involves the estimation of "reduced-form" price equations. A typical reduced-form model might explain the variation in the price of a product as a function of a series of variables relating to cost, demand, and market structure. In some cases, the model would include additional "dummy variables" (i.e., variables that take on the values zero or one) that

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2. *See, e.g.*, Alabama v. Blue Bird Body Co., 573 F.2d 309 (5th Cir. 1978); *In re* Domestic Air Transp. Antitrust Litig., 137 F.R.D. 677, 683 (N.D. Ga. 1991). Another important application of quantitative methods comes in the class certification arena. Class certification requires plaintiff to show, among other things, that there are common legal and factual issues among the members of the class. Typically, this involves a showing by plaintiff that damages can, in principle, be evaluated through a common method, and that the named plaintiff adequately represents the members of the class. *See, e.g.*, *In re* Polypropylene Carpet Antitrust Litig., 996 F. Supp. 18, 29 (N.D. Ga. 1997); *In re* Terazosin Hydrochloride Antitrust Litig., 280 F.2d 124 (11th Cir. 2001); Bradburn Parent/Teacher Store v. 3M, No. 02-7676, 2004 WL 1842987 (E.D. Pa. 2004).

3. *See, e.g.*, United States v. Eastman Kodak Co., 853 F. Supp. 1454 (W.D.N.Y. 1994). It is difficult to see how quantitative evidence alone could provide conclusory evidence about conspiracy. However, such evidence could support a view that alleged conspiratorial behavior was (or was not) consistent with the unilateral self-interest of the firms alleged to have conspired. *See, e.g.*, Ohio *ex rel.* Montgomery v. Louis Trauth Dairy, 925 F. Supp. 1247 (S.D. Ohio 1996).

4. *See, e.g.*, *Polypropylene Carpet*, 996 F. Supp. at 22-29.

5. *See, e.g.*, Colorado v. Goodell Bros., No. 84-A-803, 1986 WL 5073 (D. Colo. 1987). In addition to class certification, this chapter also does not address questions relating to the admissibility of expert testimony.

represent geographic or time differences in prices that account for variables omitted from the model (the set of such variables are the "fixed effects"). The model is called "reduced form" because the price equation is derived from other more basic economic relationships relating to demand and supply. As a result, the parameters (the variable coefficients in a multiple regression model) of a reduced-form equation are typically themselves functions of a number of the structural parameters (the parameters of the underlying economic relationships).

Reduced-form relationships are frequently easier to estimate than the structural relationships from which they are derived. It can be difficult to identify demand, for example, both conceptually (due to an inability to distinguish the demand from supply forces) and empirically (due to a lack of data). There are many occasions in which reduced-form estimates can help to answer relevant questions. However, there are risks associated with the use of reduced-form models. While the parameters from structural equations are linked to the underlying economics, reduced-form parameters are often not. As a result, one runs the risk of generating misleading results when the nature of competition changes over time.

The reduced-form model. A typical reduced-form multiple regression equation might take the following form:

$$P_{it} = \alpha + \beta w_{it} + \gamma y_{it} + \delta s_{it} + \varepsilon_{it} \tag{1}$$

In this notation, P_{it} represents the price of a product at time t paid by customer i (or, in many specifications, in region i), w is a set of variables that affect per-unit costs (e.g., input prices), y is a group of variables affecting demand (e.g., the prices of substitute products), and s is a set of variables related to market structure (e.g., seller concentration).

In Equation (1), the observations are drawn from a panel that reflects both cross-section and time-series variation. The linear form can account for models that are explicitly linear or that can be linearized after a transformation of the variables (e.g., the log-linear model). The error term can be seen to reflect random shifts in demand, marginal cost, or conduct by market participants. Typically, the error is assumed to be independent of, and therefore uncorrelated with, all of the right-hand variables. For example, an increase in the firms' costs of production not reflected in the included cost variables may cause price to increase, but it is assumed that the resulting price increase will not in turn affect market structure.[6]

The model in Equation (1) is termed reduced-form because it describes the equilibrium price that results from the interaction of demand and supply (or cost) forces in an industry, with the output variable having been removed by substitution. The cost and demand-shift variables included in these regressions are typically viewed as exogenous, since they are presumed to be determined independently of the dependent variable and therefore unaffected by it.[7] Variables related to market demand appear in

6. This presumes that the model does not contain a lagged dependent variable and a serially correlated error.

7. For a discussion of the possible endogeneity of the market concentration variable, see William Evans, Luke Froeb & Gregory J. Werden, *Endogeneity in the Concentration-Price Relationship: Causes, Consequences, and Cures*, 61 J. INDUS. ECON. 431 (1993).

the reduced-form price equation for two reasons. First, shifts in demand may alter marginal cost by changing the scale of firm operations. Second, oligopolists have an incentive to increase their markup of price over marginal cost when demand becomes more inelastic. Consequently, variables related to market structure may appear in the reduced-form price equation because they reflect the extent to which the firms are able to exercise market power.

Reduced-form models are attractive in part because their modeling and data requirements are manageable. They can help to answer questions such as whether prices were higher during the period of alleged conspiracy, even when there is not sufficient information to isolate the structure of demand and supply separately. They make only limited computational demands and can often be estimated (for single-equation linear models) using the ordinary least squares methodology.[8]

When possible and appropriate, economic theory can be used to assist in the specification of the underlying structural relationships. Typically, theory will inform the question of what variables to include in a regression model, but it is rare that theory will help one to decide the particular functional form to prefer for a model. As a result, the manageability of reduced-form models may come with a cost. First, the omission of relevant variables can bias the results. If, for example, costs were high during those periods of alleged wrongful behavior because of the influence of variables not included in the regression model, or if demand grew more inelastic during that period in ways not captured by the included demand-side variables, then a dummy variable reflecting the likely effect of wrongful behavior might have a large positive coefficient for reasons unrelated to the existence of the alleged conspiracy. Second, the results might not be robust to the choice of functional form. For example, a dummy variable indicating the likely effect of a price-fixing conspiracy might have a large positive coefficient, but the coefficient might be reduced when a log-linear model is estimated.[9] As a general rule, any inference that anticompetitive behavior led to supracompetitive prices can be made with greater confidence if the measure of the difference between average prices in the conspiratorial period and the benchmark period remains large irrespective of the functional form chosen.

There are occasions in which regression models can be useful if one wishes to simulate the likely effects of potentially anticompetitive behavior. While merger simulation is perhaps the best known application in antitrust, simulation methods have been used in a variety of other contexts.[10] As a general rule, however, simulations that use reduced-form rather than structural price equations may not be reliable if the underlying structural parameters would be different in the but-for world. This could be a problem, for example, if one were using simulation to predict the price that buyers

8. *See, e.g.*, ROBERT S. PINDYCK & DANIEL L. RUBINFELD, ECONOMETRIC MODELS AND ECONOMIC FORECASTS (4th ed. 1998).

9. The term "large" is used to suggest practical economic significance, that is, a magnitude that matters economically. In contrast, statistical significance characterizes the precision with which a parameter or parameters are measured.

10. For a review of merger simulation methods, see ABA SECTION OF ANTITRUST LAW, *supra* note 1. Some alternative uses of simulation are discussed in Rubinfeld, *Courtroom*, *supra* note 1.

would have obtained had firms not conspired. Buyers that are aware (at least probabilistically) about a possible violation may view any expected damage recovery as a price reduction. A simulation that does not account for this possibility may overstate the magnitude of the overcharge to buyers, though this possibility depends heavily on buyers having had knowledge of the violation and its likely prosecution.

Pass through. Reduced-form methods can be used to identify the rate at which a firm has historically passed through firm-specific cost changes to prices.[11] Pass-through issues are often central in private indirect purchaser cases, where it is important to evaluate the extent to which any direct purchaser overcharges are passed on to indirect purchasers.[12]

Whatever the application, the pass-through rate is an important element in an assessment of the net effect of the transaction on prices paid by buyers. For example, if direct customers were overcharged by 10 percent on their purchases, and it is determined that the pass-through rate is 30 percent, then indirect purchasers will have damages of 3 percent, rather than 10 percent. Pass-through rates depend crucially on the nature of competition, the market shares of the firms that are affected by the wrongful activity, and on the shapes of the relevant demand and cost curves.[13]

Inferences about the firm-specific pass-through rate can be made by estimating a reduced-form price equation that relates a firm's price (p) to its own costs (c) and its rival's costs (cr) and a series of fixed effects variables (F), as in the following equation (from which firm and time subscripts have been omitted):

$$p = \alpha + \beta_1 c + \beta_2 cr + \lambda F + \varepsilon \tag{2}$$

In this model, the competitor's cost variable can be thought of as a proxy for industry-wide costs. With industry-wide costs included, the cost variable picks up only the effect of firm-specific cost variation on prices.[14]

It is important to understand with respect to mergers that the firm-specific pass-through rate may not be the appropriate rate to apply to the efficiencies that would be achieved, given that the merger might change the extent to which the firms competed. In *Staples*, this did not turn out to be a problem, since the inclusion of variables related

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11. Perhaps the best example came out of the Federal Trade Commission's *Staples* litigation. FTC v. Staples, Inc., 970 F. Supp. 1066 (D.D.C. 1997); *see also* ABA SECTION OF ANTITRUST LAW, *supra* note 1. *See generally* Orley Ashenfelter et al., Identifying the Firm-Specific Cost Pass-Through Rate (Federal Trade Comm'n, Working Paper No. 217, January 1998).

12. *See, e.g.*, Ronald Cotterill et al., *Beyond Illinois Brick: The Law and Economics of Cost Pass-Through in the ADM Price Fixing Case,* 18 REV. INDUS. ORG. 45 (2001); Robert Taylor, *Indirect Damages from Price Fixing: The Alabama Lysine Case*, 18 REV. INDUS. ORG. 33 (2001).

13. If the market is otherwise competitive, the pass-through fraction is given by $Es/(Es – Ed)$, where Es is the elasticity of supply and Ed the elasticity of demand. *See* ROBERT S. PINDYCK & DANIEL L. RUBINFELD, MICROECONOMICS 328 (6th ed. 2005). The authors point out that it is quite possible that pass-through rates will be lower in industries in which firms have substantial market power than in more competitive industries.

14. In *Staples* the court agreed with the Federal Trade Commission's expert economist that Staples had historically passed through only 15% of firm-specific cost reductions to consumers. *See Staples*, 970 F. Supp. at 1090.

to market structure in the regression model did not substantively change the regression results.[15]

Summary. Reduced-form models have the potential to provide evidence on a wide range of problems. They offer relatively straightforward approaches that can be explained with relative ease to triers of fact. Nevertheless, reduced-form methods raise a diverse array of issues, including, but not limited to, the possibility of omitted variables, errors in data measurement, and the potential endogeneity of one or more explanatory variables.

Reduced-form models are least desirable when the key question for litigation depends on structural parameters, which can be difficult to recover from reduced forms. Accordingly, the next subsection discusses the use of structural models, particularly those aimed at identifying the structure of demand.

2.2. Structural models

Whether there is sufficient demand substitution to prevent the exercise of market power depends on the extent to which consumers will be diverted to other products in the face of a price increase (as measured by the price elasticity of demand for the product). Thus, identifying the structure of the demand for products is central to the analysis of market definition. The price elasticity of demand measures this directly. However, it is sometimes useful to identify the particular set of products to which lost sales are likely to be diverted; these diversion calculations depend crucially on the various cross-price elasticities of demand. Cross-price elasticities are often central to the identification of localized competition in evaluating the unilateral competitive effects of mergers.

The extent and nature of demand substitution can be determined in a number of ways. This section focuses on three methods: the empirical estimation of demand elasticities, the use of transactions or bidding data to learn about the structure of preferences, and the use of survey techniques.

Demand elasticity estimation. Typically, demand elasticities are estimated using inverse demand functions, with price being a function of output and other variables (geographic area and time subscripts have been omitted):

$$p = \alpha + \beta q + \delta qs + \gamma y + \varepsilon \tag{3}$$

In Equation (3), the price of a product is thought to depend on a vector of quantities sold (q), the quantity of sales of substitute products (qs), and a group of demand variables (y). Equation (3) presumes that observations are drawn from multiple markets (such as geographic areas) and at various periods of time (typically weeks, months, quarters, or years). The own- and cross-elasticities of inverse demand are obtained from the estimates of the β's and δ's.

In some antitrust applications, it is useful to estimate residual demand functions (rather than the more traditional Marshallian (structural) demand functions). The residual demand function omits the quantity variables for products other than the one of

15. Ashenfelter et al., *supra* note 11.

interest and adds a vector of cost-shift variables thought to affect the price and output of the omitted products.[16]

Whether one should prefer structural or residual demand elasticities depends on the question asked and the available data. The elasticities of the structural demand function provide information about buyer preferences; they show which products are close substitutes and which are not. More specifically, they summarize the way buyers would respond to price or output changes, assuming that rival firms do not change their decisions. Either approach can provide information relevant to identifying single firm market power or to determining whether a merger between sellers of differentiated products would permit the merged firm to exercise market power unilaterally.

Reduced-form models are particularly useful when one believes the underlying economic structure of a market is unlikely to be affected by a merger or to have been unaffected by alleged anticompetitive acts. Thus, in *In re Polypropylene Carpet Antitrust Litigation*, the plaintiff's economic expert, Martin Asher, proposed a damage model in which price was a function of cost factors, demand variables, market structure variables, and a set of dummies that accounted for fixed effects (price differences associated with different means of distribution).[17] The case for a reduced form was based on the view that all of the explanatory variables were reasonably viewed as exogenous (i.e., determined independently of the dependent variable and therefore unaffected by it).

Structural models are often useful when one is estimating a demand system that is to be used to simulate the likely effects of a merger, especially one in which the merger itself may affect the nature of competition in the relevant market. For example, the econometrics underlying the analysis of the likely effects of the merger of two cereal companies, Post and Nabisco, was debated at trial using structural models of demand.[18]

When using statistical methods to uncover information about relevant demand elasticities, it is important to run through a checklist of possible issues. Some of the more important issues are discussed below.

Identification. Price and output are "endogenous" variables, i.e., they are jointly determined by the intersection of demand and supply. As a consequence of the presence of an endogenous variable on the right-hand side of a demand equation, ordinary least squares estimation of Equation (3) is likely to yield biased estimates of own- and cross-elasticities of demand. The identification issue relates to whether or not variations in price and output are due primarily to shifts in supply; without further information that allows one to distinguish demand from supply, demand will not be identified.

To identify demand, one should look for "natural experiments," i.e., occasions in the past in which exogenous cost shocks have affected the industry under study. Variables that reflect cost changes, such as the prices of key production inputs, provide the most

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16. *See* Jonathan B. Baker & Timothy F. Bresnahan, *Estimating the Residual Demand Curve Facing a Single Firm*, 6 INT'L J. INDUS. ORG. 283 (1988).
17. *Polypropylene Carpet*, 996 F. Supp. at 29. For a more detailed discussion, see Baker & Rubinfeld, *supra* note 1.
18. *See* Daniel L. Rubinfeld, *Market Definition with Differentiated Products: The Post-Nabisco Cereal Merger*, 68 ANTITRUST L.J. 163 (2000); *see also* Aviv Nevo, *Mergers with Differentiated Products: The Case of the Ready-to-Eat Cereal Industry*, 31 RAND J. ECON. 395 (2000).

natural instruments for isolating demand functions. It is sometimes the case that one can obtain information about cost-shifting for all of the products at issue. Then identification and estimation of demand are likely to be straightforward. Often, however, there is insufficient cost-shifting information. In this case, identification is possible only if one makes additional assumptions about the nature of the demand.[19] When a demand function cannot be identified econometrically, the questions that can be answered empirically may be limited to those for which reduced-form methods provide an answer.

One common identification strategy when the number of cost-shift variables is small is to restrict the parameters of the demand system, for example by constraining all products in a group to enter the demand system with a common parameter or by imposing symmetry on the cross-elasticities. If the number of free parameters is limited, then even a handful of cost-shift variables may be sufficient to achieve identification.

The strategy of restricting the parameters of the demand system is almost always adopted when estimating demand functions for individual goods in differentiated products markets with a substantial number of products. The analysis is difficult because any increase in prices of the products of the firms that merge can induce increases in rivals' prices, and because the analysis of these effects can be sensitive to assumptions about the form of demand functions (e.g., Is the function form linear or nonlinear?), demand symmetries (e.g., Are cross-price elasticities symmetric?), the nature of the interaction among firms (e.g., Is competition Bertrand, Cournot, Stackleberg, other?) and the possible presence of economies of scale and scope. Moreover, in consumer cases, the analysis may also be sensitive to the assumptions that are made (or not made) about the timing of purchasers. For example, failure to account for inventorying of consumer goods could affect one's estimates of demand parameters.[20]

A further complication arises because firms selling branded consumer products may not compete solely on price. Marketing variables such as advertising and promotion can affect buyer substitution patterns. If these variables are important in buyer decision making, any available relevant data should be incorporated into the analysis of demand.[21]

With numerous products to be taken into account, any empirical analysis must make some assumptions that simplify the nature of the demand for the products that are being studied. While the particulars differ, it is typical to assume that some cross-price elasticities are zero and that there is symmetry in the nonzero cross-price elasticities. One way to achieve these restrictions is to characterize demand decisions as following a multilevel decision-making process and to aggregate individual brands into sensible

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19. *See* Timothy Bresnahan, *Empirical Methods for Industries with Market Power*, *in* 2 HANDBOOK OF INDUSTRIAL ORGANIZATION 1011, table 1 (Richard Schmalensee & Robert Willig eds., 1989) (listing eight possible identification strategies in this setting).

20. *See* Baker, *supra* note 1, at 352-55.

21. For a discussion of the importance of accounting for marketing issues, see Daniel Hosken et al., *Issues in Scanner Data*, *in* ECONOMETRICS: LEGAL, PRACTICAL, & TECHNICAL ISSUES at app. IV (John Harkrider ed., 2005).

aggregates, assuming that the demands for products in one nest or segment of a larger demand structure are separable from the demands of products in other nests.[22]

To illustrate the nesting concept, one might think of the decision to obtain a drug prescription as occurring in the third stage of a three-stage decision-making process. The first stage might allocate consumer health expenditures into prescription drugs, physician costs, and other health-related expenses. The second stage might then determine the demand for various drug categories, such as anti-infectives or anti-inflammatories, while the third stage would explain allocations among particular drugs within a category. The multistage model would greatly reduce the choice among many, many individual drugs into a choice among a relatively small number of nests, and then, with the aggregation of drugs into nests, would drastically reduce the number of elasticities that must be estimated.

The benefits of reducing the number of parameters are substantial—it facilitates identification when instruments are few, simplifies computations, and deals with limited data. However, the results can be quite sensitive to the restrictions that are made. In particular, the decision to include a product or group of products in one nest rather than another can substantially affect the conclusion that one reaches concerning the definition of the relevant antitrust market.[23]

Hausman and Leonard have proposed a different solution to the identification problem, one that employs the nationwide component of individual city prices as an instrumental variable for identifying demand.[24] The authors assume that whatever the underlying structural model, the reduced-form price equation for an individual product sold in city i in period t is

$$P_{it} = \alpha_{it} + \beta c_t + \varepsilon_{it} \tag{4}$$

where c_t is the marginal cost in period t, which is assumed to be substantially the same across geographic areas. While demand and cost shocks might affect the error term, Equation (4) assumes that there are no nationwide demand-shift variables. In this case, the nationwide component of price will reflect the effect of changes in cost, but not the influence of demand. As a result, this price component can serve as a suitable instrument to identify demand.

This instrumental variables approach is relatively easy to use if one has access to scanner data. However, one must believe that shocks to the nationwide component of prices in the industry during the time period under study result mainly from variations in cost, not demand. In some cases, identification may flow directly from the fact that prices affect demand with a lag; with sufficient lags, there will be little chance that prices and quantities will feed back on each other sufficiently to make price endogenous. Here, as in many other ways, the nonstatistical evidence available in the antitrust

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22. *See, e.g.*, Jerry A. Hausman, Gregory K. Leonard & J. Douglas Zona, *Competitive Analysis with Differentiated Products*, 34 ANNALES D'ECONOMIE ET DE STATISTIQUE 159 (1994); *see also* Nevo, *supra* note 18.

23. *See* Daniel L. Rubinfeld, *supra* note 18.

24. *See* Hausman & Leonard, *supra* note 1.

litigation context can complement the statistical evidence and make expert testimony more compelling.

Functional forms. An additional methodological question involves the choice of functional form. The functional form describes the shape of the regression "line."[25] While a wide variety of functional forms are possible, the most common forms are linear (a straight line in a two variable regression model) and linear in the logarithms or log-linear (curvilinear in a two variable regression model). The linear regression model is sufficiently flexible to account for both linear and log-linear specifications. Both are manageable and allow for the calculation of own- and cross-price elasticities, but each constrains the way in which demand elasticities change as price changes. To investigate statistically the extent to which demand elasticities change with price, more flexible demand systems may be desirable.[26]

There is a cost to having greater flexibility in functional forms. To achieve flexibility in an economical manner, it may be necessary to impose other constraints, such as those built into a multistage demand system. Frequently the tension between the desire for functional form flexibility and the need to restrict parameters can best be addressed by incorporating restrictions that facilitate estimation, while testing the sensitivity of the results to alternative plausible assumptions.

The reliability of simulations. In dealing with unilateral effects, for example, it is important to evaluate the magnitude of the incentive of the merging parties to raise price after the merger. The elasticities of demand themselves may provide indicators of the strength of these incentives, or they may be combined with information or assumptions about cost and oligopoly behavior to simulate the effect of the merger on price.[27] Under some circumstances, the simulated price increases may be more informative indicators of the strength of merged firm's incentive to raise price than what one can tell from demand and cross-elasticities themselves. Simulations also provide a valuable method of assessing the sensitivity of predictions to uncertainty in parameter estimates and to alternative assumptions about the underlying demand and supply functions and market structure.

Simulation has its own risks. To use the simulation approach, one must make assumptions about how costs, demand, and the nature of oligopoly behavior are likely to change postacquisition.[28] These difficulties may mean that in some cases complex simulations will contribute little more than can be learned about the anticompetitive incentive of the merging firms to raise price from the demand elasticities alone.

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25. If there is more than one explanatory variable, the regression will be characterized by a plane.

26. One popular flexible system (which may itself not be flexible enough) is the AIDS (almost ideal demand system). *See generally* Angus Deaton & J. Muellbauer, *An Almost Ideal Demand System*, 70 AM. ECON. REV. 312 (1980).

27. *See, e.g.*, Hausman & Leonard, *supra* note 18; Nevo, *supra* note 18; Gregory J. Werden, *Simulating the Effects of Differentiated Products Mergers: A Practical Alternative to Structural Merger Policy*, 5 GEO. MASON L. REV. 363 (1997).

28. *See generally* Baker, *supra* note 1, at 356-60. With price-cost margin data and an assumption of Bertrand pricing, one can estimate own- and cross-elasticities of demand. Alternatively, if demand elasticities are estimated one can estimate marginal costs.

2.3. Other methods for identifying the structure of demand

In some instances, direct estimation of demand elasticities will not be possible. There are, however, alternative statistical methodologies that can be useful if one wishes to learn about consumer preferences: (1) the use of auction models to make inferences from bidding records, (2) inferring preferences from an analysis of the attributes of goods and services actually purchased by consumers, and (3) eliciting preferences from survey responses to hypothetical questions involving product attributes.

Auction models. In some cases, choices are made through a bidding process, whether through formal auctions or informal bidding. Econometric tools have been developed to analyze auctions generally, and in particular, to distinguish bid rigging from competitive bidding in formal auctions. One interesting application involves an attempt to identify phantom bidding by cartel members (noncompetitive bids submitted by firms feigning competition). The approach involves estimating a regression model that explains the bidding behavior of firms that are assumed not to be involved in the bid rigging, using the resulting estimated bid functions to predict the "competitive" bidding of those firms alleged to be involved in bid rigging, and comparing the predicted bids to the actual bids. Deviations that are otherwise consistent with a bid-rigging theory can be taken as support for the presence of a bid-rigging conspiracy.[29] The method requires accurate measurement of the variables that are expected to predict differences in behavior across sellers in the absence of bid rigging, such the differences in the costs of serving various buyers.[30]

Auction modeling can also be useful in a merger setting, when one wishes to evaluate likely unilateral effects. One approach is to estimate parameters that describe the auction process in a differentiated oligopoly. These parameters can then be used to predict the hypothetical effects of a merger, as, for example, in the analysis of Oracle's acquisition of PeopleSoft.[31]

Auction models may also be useful in informal bidding situations when there are not sufficient market data to estimate demand functions directly, but it is possible to find out a good deal about the winning and losing bids.[32] The information about the bids that are made—especially winning bids—can be used to infer preferences. For example, one

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29. This approach requires that the value of the contract or procurement to bidders be based on observable bidder characteristics or be identical across the firms. For applications to the bidding process for highway paving jobs on Long Island in the early 1980s and to the procurement process for Ohio school milk auctions throughout the 1980s, see Robert H. Porter & J. Douglas Zona, Ohio School Milk Markets: An Analysis of Bidding (Apr. 1997) (unpublished article, on file with author); Robert H. Porter & J. Douglas Zona, *Detection of Bid Rigging in Procurement Auctions*, 101 J. POL. ECON. 518 (1993).

30. A somewhat different approach is taken in Luke M. Froeb, Robert A. Koyak & Gregory J. Werden, *What is the Effect of Bid-Rigging on Prices?*, 42 ECON. LETTERS 419 (1993). Rather than assume stability between the conspiracy and nonconspiracy periods, the authors alternatively backcast but-for prices from the nonconspiracy period into the conspiracy period and forecast but-for conspiratorial prices from the conspiracy period into the nonconspiratorial period.

31. The testimony of the government's expert, Preston McAfee, was given relatively little weight in the court's decision against the government because it was arguably predicated on the wrong market definition. United States v. Oracle Corp., 331 F. Supp. 2d 1098 (N.D. Cal. 2004).

32. *See, e.g.*, Rubinfeld & Steiner, *supra* note 1.

can regress the winning bid in an informal bidding competition against factors that would affect the individual sellers' bids (e.g., estimates of individual seller costs, volume of product to be sold, delivery conditions). The resulting regression (whose functional firm might ideally be suggested from information about the structure of the underlying market) could then be used to evaluate whether the merging firms are or are not close substitutes (e.g., similar bidding patterns could reflect similar costs). This information can, in turn, be used to predict the price effects of a proposed merger.

Another option is to analyze the choice of firms that were invited to bid by a seller involved in procurement bidding. Given that procuring bidding is relatively costly, one would expect that only those bidders that offer relatively close alternatives will be invited to bid. One would then presume that the more frequently that pairs of firms bid, other things equal, the more similar the products. An analysis of the closeness of products could be achieved by evaluating whether the probability that one firm will bid in a particular auction conditional on a second firm's bidding is significantly different from the unconditional probability. A higher conditional probability would support the view that the two products are relatively close substitutes. Alternatively, one could use multiple regression analysis to evaluate bids, holding constant factors that are believed to reflect the degree of substitutability of the products.

Consider, for example, the Department of Justice's 1998 investigation of two proposed accounting firm mergers: Coopers & Lybrand and Price Waterhouse, and Ernst & Young and KPMG Peat Marwick.[33] Auditing firms are typically chosen through an informal bidding process. In the market for auditing services, accounting firms routinely develop industry specializations through industry-specific investments in software, personnel, and marketing. This raises the possibility that the merger of two firms with expertise in the same or similar markets could lead to higher auditing fees.

The Department of Justice analysis of the two accounting mergers relied in part on a historical data set on the audit fees and audit bids for each of the then big six accounting companies.

The analytical approach involves estimating a price equation that explains audit fees as a function of audit costs, client characteristics (e.g., sales volume, assets, costs, industry), and a measure of the prior years' market shares of the firm in the client's industry. The regression parameters could serve as the basis for a test of whether auditors have valuable expertise in auditing clients in particular industries and a simulation of the likely price effects of merger.

Auction techniques have their own limitations. As a general rule, their predictions can be quite sensitive to the assumption made about differences in costs, if any, among competing bidders, and to the assumption made about the nature of competition in the market (the form of auction, one shot or repeated play, etc.). Furthermore, it can be difficult to provide statistical measures that characterize the reliability of the resulting predictions.

Market shares. Another approach to understanding the nature of demand involves uncovering the valuation buyers place on individual product characteristics from the

33. *See* Mary W. Sullivan, The Effect of the Big Eight Accounting Firm Mergers on the Market for Audit Services (U.S. Dep't of Justice Economic Analysis Group, Working Paper No. EAG 00-2, 2000).

distribution of buyers' first choices (i.e., market shares).[34] Each buyer is assumed to select the product that gives him the most value, taking into account product characteristics, including price. The distribution of buyer preferences is then described by relating market shares to observable product characteristics, given an assumption about the distribution of unobservable buyer tastes. Typically, the number of characteristics is smaller than the number of products, so that the number of parameters to be estimated is substantially less than the number of own- and cross-elasticities of demand.

In this framework, the parameters of the demand functions are selected to make the distribution of buyers' first choices fit market shares as closely as possible. The distribution of choices, in turn, restricts the way in which products substitute for each other. Using this approach, instruments for price are required in order to generate consistent parameter estimates;[35] this is analogous to the problem of identifying demand when product quality varies. Natural instruments are input prices or variables related to the degree of rivalry among the firms. With consistent estimates of the parameters of the distribution of preferences, it is possible to derive implied demand cross-elasticities between, for example, the products of merging firms.

This approach builds in the reasonable assumption that when individuals substitute away from a good whose price has increased they will substitute towards goods with similar characteristics. To work effectively, however, the demand function must be flexible in form, sufficiently flexible to capture the manner in which preferences depend on product characteristics. As usual, there is a trade-off. Estimating additional parameters permits greater flexibility, but the computational difficulties can be severe, and the results can be sensitive to the specific methodology used to estimate the model. As with other methodologies, it is important to evaluate the robustness of the empirical results.

Conjoint survey methods. While survey methods have been widely utilized in litigation involving issues such as false advertising and copyright and patent infringement, their use in antitrust remains rare.[36] A properly designed survey (including the statistical design of the research and the crafting of the survey instrument) can avoid a number of problems that are inherent in market-based data. Thus, the judicious design of the survey questions and the selection of those to be surveyed can ensure that explanatory variables are exogenous, rather than endogenous.

Surveys are sometimes treated with skepticism when they rely on answers to hypothetical questions in order to predict but-for behavior. While one must be careful in

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34. For further discussion and references to an extensive bibliography, see Steven T. Berry, *Estimating Discrete-Choice Models of Product Differentiation*, 25 RAND J. ECON. 242 (1994).

35. The academic literature implementing this methodology employs a generalized method of moments (GMM) estimation technique, a natural choice for fitting sample moments to a distribution. Least squares regression can be thought of as a special case of the GMM technique, just as it can be thought of as a special case of the more familiar maximum likelihood estimation approach. *See* PINDYCK & RUBINFELD, *supra* note 8, ch. 10.

36. See Shari Seidman Diamond, *Reference Guide on Survey Research*, *in* REFERENCE MANUAL ON SCIENTIFIC EVIDENCE 229 (2d ed. 2000), for an overview of the use of survey methods and its many applications.

interpreting survey answers when respondents are not faced with choices that involve real economic constraints, an appropriate survey approach can be used to predict the price effects of a merger or to evaluate the extent of harm caused by a firm's exclusionary practices. "Conjoint analysis" is a potentially useful tool. It has proven valuable as a marketing tool for deciding whether to match a competitor's price increase, for pricing new brands, and for setting new prices among the bundle of existing brands.[37]

To use the conjoint approach to evaluate the demand for a group of products, one might show a group of respondents descriptions of actual and hypothetical characteristics of each of the products at issue (including price). Respondents would be asked to allocate a fixed number of points among each set of product options. The responses would be taken as measures of the likelihood that individuals will make actual consumption choices, conditional on the prices and product characteristics that they face. With a judicious choice of product descriptions, consumer preferences can be evaluated using a discrete choice demand model. A successful conjoint analysis chooses an experimental design with a subset of all possibilities that is sufficient to estimate underlying consumer demand functions with reasonable accuracy.[38]

Conjoint analysis can provide empirical answers to questions for which alternative approaches would fail because of a lack of data. Moreover, because it is based on an experimental design, it is subject to statistical testing. There are, of course, a number of risks. First, because the answers to hypothetical questions may not accurately reflect the choices that individuals would make under actual market conditions. Second, because the empirical methodology usually relies on the aggregation of responses, some information about individual preferences may not be utilized. Third, because individual responses do not flow from a utility-maximizing constrained optimization problem, the predicted but-for prices may not have a high degree of accuracy. Finally, if one wishes to account for interactions among product characteristics, one must use a complex experimental design. This may substantially add to the cost of the study.

The government has relied on conjoint studies on a number of occasions. In reaching a consent decree with a number of ski resort operators in 1997, conjoint analysis played an important role.[39] More recently, conjoint methods were used in an exclusionary practices case: *United States v. Dentsply*.[40] In *Dentsply*, the government alleged that the defendant, which manufacturers artificial teeth for use in dentures and other restorative appliances, had engaged in a series of restrictive dealing relationships that allowed it maintain its monopoly power in the sale of prefabricated artificial teeth to dealers and laboratories. Because the conjoint methods were excluded as evidence by the district

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37. *See, e.g.*, Paul E. Green & Jeffrey Savitz, *Applying Conjoint Analysis to Product Assortment and Pricing in Retailing Research*, 2 PRICING STRATEGY & PRAC. 4 (1994).

38. For a specific application, see Vijay Mahajan, Paul E. Green & Stephen M. Goldberg, *A Conjoint Model for Measuring Self- and Cross-Price/Demand Relationships*, 19 J. MARKETING RES. 334 (1982).

39. Competitive Impact Statement, United States v. Vail Resorts, 62 Fed. Reg. 5037, 1997 WL 36727 (D. Colo. 1997) (No. 97-B-10), *available at* http://www.usdoj.gov/atr/cases/f1000/1014.htm.

40. United States v. Dentsply Int'l, 277 F. Supp. 2d 387 (D. Del.), *rev'd*, 399 F.3d 181 (D. Del. 2003); *see also* Competitive Impact Statement, *supra* note 39.

court, all of the issues surrounding their ability to provide accurate estimates of the extent of exclusion were not fully debated.

2.4. Summary

A variety of methods that allow one to identify the structure of demand have proven to be extremely valuable in antitrust analysis. Once one has identified the shape of demand functions, it is possible to evaluate questions relating to market and monopoly power and to estimate the price effects of a merger or of alleged anticompetitive behavior. As a general rule, the methods that have been discussed are not as straightforward as reduced-form methods, and they typically require more data. But in many cases they can generate answers to questions that reduced-form methods cannot address. Even when both reduced-form and structural methods would be informative, the latter can often exploit qualitative information about market structure to improve the precision of parameter estimates and treat problems arising from the potential endogeneity of price and/or output variables.

3. Applications

3.1. Liability issues

In determining whether or not a violation has occurred, hypothesis testing can be a statistical tool. Hypothesis testing involves specific statistical tests which allow one to draw inferences about which of two or more competing explanations of an agreed set of events is the most credible. A typical hypothesis test will begin with a null hypothesis of no violation, with the alternative being that there was a violation. The tools of classical statistics and econometrics are well adapted to the use of such testing procedures.[41] However, there are particular issues that are likely to arise in the context of antitrust litigation that merit special attention.[42]

The alternative hypothesis. Assume for purposes of discussion that the issue in question is whether or not a manufacturer's distribution arrangement is anticompetitive. To some extent, perhaps indeed a great extent, the answer to the question of whether there is liability will depend not only on the law but also on range of qualitative evidence that flows from depositions, documents, and other traditional types of evidence. However, liability issues can also be informed by quantitative studies.

A valid study begins with an appropriate model—a characterization of the economic forces at issue. A typical model would include a dependent variable, such as the wholesale price of the product in question. The model will also include an explanatory variable that allows for the correct evaluation of alternative hypotheses. For example, the variable of interest might be a dummy variable that takes on the value 1 when the new distribution system was in place and 0 otherwise.

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41. *See* David Kaye & David Freedman, *Reference Guide on Statistics*, *in* REFERENCE MANUAL ON SCIENTIFIC EVIDENCE 83 (2d ed. 2000); Daniel L. Rubinfeld, *Reference Guide on Multiple Regression*, *in* REFERENCE MANUAL ON SCIENTIFIC EVIDENCE 179 (2d ed. 2000).

42. These issues are spelled out in detail in Rubinfeld, *Courtroom*, *supra* note 1.

Suppose that the following model has been specified:

$$P_{it} = \alpha + \beta D_{it} + \gamma X_{it} + \varepsilon_{it} \tag{5}$$

where P is the price of a product, D is the dummy variable that indicates the period of alleged violation, and X represents a group of additional explanatory variables that might support alternative substantive hypotheses that can be accounted for by the regression analysis (the subscript t represents a time dimension and the subscript i represents customers). One might then put forward the null hypothesis that $\beta = 0$, namely, that the period of alleged violation had no effect on price. A positive and statistically significant coefficient on the D variable would support a finding of liability.

If this hypothesis testing approach is to be used, it is important to include in the model variables that are thought to influence the dependent variable. To see this, note that if there are no X variables in the model, then the only alternative to a finding that there is no difference in the product price between the actual and but-for periods is a finding of liability. The inclusion of additional explanatory variables amounts to allowing the model to incorporate alternative hypotheses other than liability.

Not all possible variables that might influence the dependent variable need be included for the analysis to be successful; some cannot be measured and others may make little difference. However, failure to include an important major explanatory variable that accounts for factors that differ between the period of alleged wrongdoing and other time periods can cause the measured effect of the dummy variable of interest to be biased, which in turn could lead to an invalid conclusion on liability. In general, omitted variables that are correlated with the dependent variable reduce the probative value of the regression analysis.

Omitting variables that are not correlated with the variable of interest is, in general, less of a concern, since the coefficient that measures the effect of the variable of interest on the dependent variable is estimated without bias. Even if bias is thought to be a risk, it may nevertheless be possible to account for bias qualitatively if one has knowledge about the relationship between the omitted variable and the explanatory variable.

Practical versus statistical significance. "Practical significance" means that the magnitude of the effect being studied is not de minimis—it is sufficiently important substantively for the court to be concerned. It is inappropriate as a general rule for liability questions to be decided solely on the basis of whether or not a particular result is or is not statistically significant. One reason is that the degree of statistical significance associated with a parameter is in part a function of the size of the sample being studied. Other things being equal, the statistical significance of a regression coefficient increases as the sample size increases. One would not want a conclusion of legal liability to rest entirely on a significance test which is sensitive to the size of the sample being studied.

Often, results that are practically significant are also statistically significant. However, it is possible with a large data set to find statistically significant coefficients that are practically insignificant. Similarly, it is also possible to obtain results that are practically significant but statistically insignificant. Suppose, for example, that a regression analysis suggests that prices are 7 percent higher in the period in which the alleged anticompetitive activity took place. If the data are such that only three of four years of data are available outside the period of alleged wrongful behavior, the 7 percent difference could be practically significant yet statistically insignificant.

3.2. Damage issues

Statistical and econometric methods can be very helpful in deciding whether a violation has led to antitrust impact, and, if so, the extent of damages (if any) that have arisen. With respect to damages, there are a number of important choices to be made in putting forward a study. Three of the most important are (1) whether the expert should rely on a structural or reduced-form model, (2) how the damage calculation should be performed, and (3) how one should test the reliability of the damage estimate.

Structural or reduced-form model? Consider a hypothetical collusive price-fixing case. The impact-damages question can be answered by comparing actual prices during the period in which wrongful conduct has occurred (which could extend beyond the period of actual wrongdoing) to the but-for prices that would arise in a world in which there had been no illegal conduct.

For such a case, the application of multiple regression would seem natural. Many things would be expected to have an effect on price in the but-for world, some through the demand side and some relating to supply. While such forces would affect price in any market, whether competitive or cartelized, they will not necessarily do so in the same way and to the same extent. To compare the actual world to the collusive world, it is helpful to use an appropriate structural model. The structural model will explain the nature of the interaction of firms in the but-for world and will specify the underlying structure of demand. It will then be (conceptually) straightforward to derive the likely effect of a price-fixing arrangement that was brought into place by a reduction in cartel output.

If there are sufficient data to allow one to estimate a structural model for the period in which there was no violation, a useful approach to damages is to estimate the structural model and to use that model to forecast what prices and quantities would be in the but-for world.[43] Damages might then be measured by the resulting lost profits on sales that were made as well as lost profits on sales that would have been made had there been no price fixing.

If such an approach is to be utilized, it is important to ask whether the other explanatory variables that are included in the regression model are known to be independent of the price-fixing activities and vice versa. If independence is not an appropriate assumption, a multiple regression analysis might reasonably account for price changes, but it would not be informative about the causal impact of the price-fixing arrangement. To see why, note that a cartel cannot only raise price and restrict output, it can also cause shifts in demand and supply that lead to price increases and take advantage of shifts in demand or supply that occur outside of the activities of the cartel. Such "indirect" effects of a cartel make statistical determination of prices but for the cartel difficult. They are especially likely to occur if demand and/or supply are heavily

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43. See Joseph E. Harrington, Jr., *Post-Cartel Pricing During Litigation*, 52 J. INDUS. ECON. 517 (2004), for an interesting application.

affected by expectations since the cartel can deliberately influence those expectations by its actions.[44]

In many circumstances, there is insufficient information to accurately characterize the underlying structure of demand or the complexity of firm interactions. The use of reduced-form models can offer a suitable, indeed often reliable, alternative. The discussion that follows focuses, for simplicity, on issues involving reduced-form models.

Suppose that one has estimated the reduced-form model in Equation (5). The coefficient on the dummy variable, D, can be taken as a measure of the difference between the average price during the cartel's existence and the average price during the but-for competitive period. The dummy variable approach is appealing because it can be applied even where there is a relative paucity of data in the nonconspiratorial period. There is, however, an important limiting assumption implicit in the dummy variable approach. That assumption is that the overall behavior of the regression model can be modeled in precisely the same way during both the conspiratorial and nonconspiratorial periods. This may be a reasonable assumption in some cases, but it should not be made without substantial thought. If prices are determined in rather complex ways, the use of a single dummy variable that will reflect mean differences between the benchmark and but-for periods may be too simplistic. In particular, if one or more of the explanatory variables in the model is correlated with the dummy variable, then the dummy variable and forecasting approaches are likely to generate different damage estimates.[45]

A more general approach, which avoids the problem just described, is the "forecasting approach." In this approach, the expert estimates a regression model for the nonconspiratorial or "benchmark" period only. That model is then used to predict or forecast but-for prices during the conspiratorial period.[46] Suppose, for example, that the model at issue involves a study of prices of time, with $t = 1$,...n the nonconspiratorial period, and $t = n+1, ...N$ the conspiratorial period. One could then use data for the first n time periods to estimate a model of the form:

$$P_t = \alpha + \gamma X_t + \varepsilon_t \tag{6}$$

and use as forecasted prices $P_f = a + cX_t$ for the time periods $n+1$ to N, where a and c are the estimated parameters of the model in Equation (6).

The forecasting approach avoids the assumption that the underlying model is identical in both the conspiratorial and nonconspiratorial periods. It is important to realize, as a consequence, that a model that fits the data well in the benchmark period may not forecast well into the period of alleged conspiracy. When using the forecasting approach, therefore, one should make an effort to report not only a measure of the

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44. For a case in which this did present a serious problem (the uranium cartel), see Rubinfeld & Steiner, *supra* note 1, at 128.

45. These issues are discussed in further detail in Fisher, *supra* note 1; Rubinfeld & Steiner, *supra* note 1; Rubinfeld, *Courtroom*, *supra* note 1; see also Richard Higgins & Paul Johnson, *The Mean Effect of Structural Change on the Dependent Variable is Accurately Measured by the Intercept Change Alone*, 80 ECON. LETTERS 255 (2003), for a more detailed evaluation of the differences between the two approaches.

46. The nonconspiratorial period could include time after as well as before the alleged conspiracy.

goodness of fit of the regression model as estimated in the benchmark period but also some measure of forecast reliability. Often, standard errors that offer information about forecast accuracy can be calculated or simulated.[47]

Are the damage estimates robust? It is essential to evaluate the reliability of any estimate of antitrust impact and/or damages. Specifically, one should ask whether the results are unusually sensitive to slight modifications of the assumptions that underlie the model and of the data itself. It is especially important to check to see if the data are themselves reliable, and if reliable, whether any regression results are highly sensitive to one or a few data points. If the assumptions of the model are valid and the data are reliable, a damage study can be highly probative, even if the results are highly sensitive. However, when the assumptions of the model are not valid, standard tests can overstate or understate the significance of the results.

An interesting case in point is offered by the controversy that arose in *Conwood Co. v. U.S. Tobacco Co*.[48] In *Conwood*, the plaintiff's expert asserted that U.S. Tobacco's illegal marketing practices had caused substantial injury. The expert put forward a "foothold" theory of damages, which presumed that in the but-for world Conwood's market share would have grown most substantially in states in which Conwood initially had a small foothold market share.

The expert's damage model used cross-section data for 50 dates and the District of Columbia (for the years 1984, 1990, and 1997). The regression, which used data on the initial market share (MS) and the change in market share (ΔMS), took the following simple form:

$$\Delta MS = \alpha + \beta * MS \tag{7}$$

The expert found a positive and significant coefficient of 0.22 on the market share variable for the 1990 to 1997 period but not for the earlier period—the coefficient was –0.13. He concluded that a change occurred after 1990, which he attributed to the illegal practices.

It is crucial to show that the results of such a damage study do not substantively change given reasonable alternative specification choices. In *Conwood*, the results were not robust. Indeed, the exclusion of a single data point for the District of Columbia (the area with the smallest sales) completely changes the results. The coefficient during the previolation period increased to 0.26, whereas the postviolation coefficient became 0.24. When one data point for the smallest area of study is dropped, there is no significant difference between the coefficient on initial market share and consequently no support for the foothold theory.

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47. See Rubinfeld, *supra* note 41, for a basic introduction and John Geweke, Daniel Houser & Michael Keane, *Simulation Based Inference for Dynamic Multinomial Choice Models*, *in* COMPANION TO THEORETICAL ECONOMETRICS 466 (Baitagi ed., 2000), for an approach to the simulation of forecast errors.

48. 290 F.3d 768 (6th Cir. 2002). The issues are described in greater detail in ABA SECTION OF ANTITRUST LAW, *supra* note 1, at 179-224, and in Motion for Leave to File Brief and Brief of Washington Legal Foundation, Stephen E. Fienber et al. as Amici Curiae in Support of Petitioners, Conwood, 290 F.3d 768 (No. 02-603), *available at* http://www.law.berkeley.edu/faculty/rubinfeldd/Profile/publications/rubinfeld_Daubert-Amicus%20Brief.pdf.

Interestingly, the jury awarded $350 million in damages, which was then trebled. However, the identification of the District of Columbia data point as an *outlier* at trial could have undermined the plaintiff's damage case.

4. Conclusion

Empirical methods offer powerful tools for understanding what has happened in the past and for simulating the likely effects of alternative scenarios. They can, in consequence, provide valuable information to courts in many areas, including class certification, the assessment of antitrust liability, the evaluation of anticompetitive harms suffered by consumers and producers, and the estimation of antitrust damages.

Because empirical analyses often appear complex to the lay person, there may be a tendency on the part of the courts or others to separate the evaluation of the evidence resulting from these methodologies from other factual evidence. Such a separation is inadvisable. The empirical analysis of data should be combined with an analysis of nonstatistical information. As more nonstatistical information is brought to bear, the systematic empirical evidence can often answer the key questions at issue in litigation more precisely. Conversely, the effort to come to grips with systematic empirical evidence often pays dividends by helping direct the search for nonstatistical evidence in documents or testimony.

It is important to use documents and deposition or oral testimony to confirm the specification of the model being utilized in an empirical study. To do so requires an appropriate mix of historical data, hypotheticals, and assumptions about behavior based on qualitative techniques. While there are important differences between the use of quantitative methods in antitrust and in the academic world, empirical antitrust analysis should draw heavily on the empirical literature in industrial organization economics. The questions raised by antitrust cases, and the data sets uniquely available under such circumstances, can provide a powerful stimulus to the development of new empirical methods, to the benefit of both the academic and the antitrust worlds.