Credit Risk Transfer and Bank Lending*

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Abstract

We study the difference between loan sales and credit default swaps. A bank lends money to an entrepreneur to undertake a positive NPV project. After the loan has been made, the bank finds out if the project benefits from monitoring and if it should sell the loan to release regulatory capital. A bank can lay off credit risk by either selling the loan or by buying credit insurance through a credit default swap. Finally, a bank holding a loan may monitor which in some cases increases the success probability. We characterize equilibria when banks choose how to lay off risk. We find that there can be excessive monitoring if there are loan sales, and insufficient monitoring if there is both an active CDS and loan sale market. We find that an active CDS market makes loan prices less informative about the success probability of the project.

Preliminary and Incomplete

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1 Introduction

In the last fifteen years, there has been a massive increase in the size of markets for credit risk. For example, a bank wishing to free up regulatory capital may sell a loan directly or buy synthetic products (credit default swaps) that effectively insure it against non-performance. We present a parsimonious model of bank lending that characterizes how the choice of these two credit risk transfer methods affects equilibrium in the loan sale and Credit Default Swap market.

Briefly, a CDS is a contract written on the solvency of a firm, (referred to variously as the “reference entity” or the “name”). If there is a credit event (default, restructuring) associated with the reference entity then the protection seller remunerates the protection buyer. In exchange, the latter pays regular premia to the former over the term of the swap. With a CDS, the originating bank retains control rights over the loan that it made. By contrast with a loan sale control rights are ceded to another.

In our model, a firm has a risky, positive NPV project and seeks funding from a bank. The bank has precise information about the project’s success probability. After it has originated the loan, the bank may be hit by a shock that leads it to lay off the credit risk from its balance sheet. There are two markets that it could use: either a CDS market in which it retains control rights, but is no longer exposed to the economic risk of the project, or a direct loan sale. In the latter case, the control rights are transferred to a third party. Ownership matters in this context because the loan’s owner may monitor the project and incur a private cost. In some cases monitoring will decrease the default probability. In this way, the originating bank’s precise information is valuable as it is required for efficient monitoring.

We characterize the equilibria that arise in the synthetic instrument (CDS) market and the loan sales market. We find that there is a tradeoff between efficient monitoring and efficient risk sharing. In particular, for the parameter range that we consider neither is simultaneously achieved. There can be too much monitoring in the presence of a CDS market and too little in the presence of only the loan sale market. In addition, we find that the presence of a CDS market makes the loan sale market less informative and increases the variance of outcomes.

Empirically, Minton, Stulz, and Williamson (2006) examine banks’ credit risk transfer practices over a sample period of 1999-2003. Of all bank holding companies with $1 billion or more assets, only 18-20 use credit derivatives during sample period however, these firms account for over 60% of sample assets. Acharya and Johnson (2006) find that information from CDS market does add to stock market information, primarily with respect to bad
news. Kiff, Michaud, and Mitchell (2002) surveys different types of CRT instruments and suggests that CDS are more pernicious than loan sales, since transactions are unobservable. Empirically, loan sales have been examined by Gupta, Singh, and Zebedee (2006), and Moerman (2005). They find that increased liquidity (lower bid-ask spreads) in secondary loan sales market lead to lower loan spreads in primary market. Gupta et al. also point out loans sold are typically senior, secured and usually sold piecemeal. Drucker and Puri (2006) present stylized facts on banks’ loan sales. Borrowers whose loans are sold are more than 1.5 times the size of borrowers whose loans are not sold. They document that those loans that are sold tend to be term loans rather than credit lines, and have more (and more restrictive) covenants than those that are not sold. They suggest that tighter covenants increase the probability of sale when (initial) lender is less reputable (lower market share, or not in top-ten lead banks). Sold loans are sold quickly: more than 60% within one month of origination, and nearly 90% within one year.

Arping (2004), Duffee and Zhou (2001), and Morrison (2005) investigate the effect of credit derivatives on relationship banking. These papers do not consider the choice between loan sales and CDS. Arping (2004) formalizes credit derivatives as a relaxation of the limited-liability constraint. By receiving negative cash flows in case of a credit event, protection sellers reinforce the bank’s incentive for efficient liquidation. Morrison (2005) models credit derivatives as the introduction of noncontractible trades between the bank and protection sellers before monitoring by the bank takes place, which may be inefficient.

2 Model

Consider the following five period model of an entrepreneur who raises funds from a bank to undertake a risky project. After the loan is originated, the bank can lay off credit risk either through a credit default swap market or a loan sales market. The owner of the loan can exert costly effort and in some cases decrease the default probability.

At $t = 0$ an entrepreneur raises money from a bank to fund a project of fixed size which pays off $R$ with probability $p$ and $C$ otherwise. The bank proposes the contract which is characterized by $R^\ell$, where $R^\ell$ is the payoff conditional on the project’s success. Thus, $R^\ell - C$ is the risky portion of the loan. Alternatively, as the size of the project is 1, $R^\ell - C$ is the yield spread on the loan.

At $t = 1$ the bank gets a private signal about the payoff to the project. With probability $\theta$ it learns that the project succeeds with probability $p + \Delta$. Thus, the bank has private information about the default probability of the project. In addition, with probability $\gamma$ the bank receives an opportunity to invest in another project correlated with the existing
one. Due to regulatory constraints, (i.e., there is a risk limit and banks have to raise costly external capital), the bank values each unit of risk eliminated (if it has a shock), at \( \beta > 0 \). We provide a specific model of regulatory constraints that generate this reduced form in Appendix 7 below.

At \( t = 2 \), the bank can off load the credit risk from its balance sheet. There are two ways in which a bank can do this, first any bank can trade in the CDS market. Second, the bank can sell the loan. If a bank enters into a credit default swap (buys protection), it buys insurance that pays off the face value of the loan if the firm defaults. It is useful to specify the cashflows of a bank who buys protection (enters into a Credit Default Swap). This contract pays off the face value of the loan if the firm defaults. This is achieved either through physical delivery, in which case the bank delivers the instrument conditional on default (valued at \( C \)) and receives \( R^e \), or through cash settlement, in which case the CDS seller pays out the amount of the contract and the bank retains the existing loan. In both cases the value of the loan plus the CDS is worth \( R^e \) conditional on default.

The originating bank may sell the loan or enter into a CDS with a risk neutral market. Prices are the expected value of the loan or the CDS contract to the market. Therefore, the market’s beliefs of the default probability are an important endogenous variable that affects the payoffs to each of these actions. Let \( p^m \) denote the market’s belief about the probability of success of the loan given that the bank participates in the CDS market, and let \( p^o \) be the market’s belief if the bank sells the loan. For notational ease we sometimes describe \( p^m = p + \theta^m \Delta \), and \( p^o = p + \theta^o \Delta \). Belief about the success probability depends on the equilibrium.

At \( t = 3 \) the agent who owns the loan can exert costly effort. To do so, he loses a benefit of \( b \). If he monitors, the probability of the project succeeding increases by \( \Delta \) if it was \( p \). Otherwise, monitoring has no benefit. In this way, information about the default probability is valuable in informing the monitoring decision. Finally, at \( t = 4 \), all claims pay off.

To resolve indifference, we assume that when banks are indifferent between laying off risk and retaining it, they retain it on their balance sheets. In addition, if a bank is indifferent
between monitoring and not, we assume that they do not monitor. In this first case, this is equivalent to an infinitesimal cost of entering a market and the second case to initiating monitoring.

2.1 Assignability

The right to assign a loan is contractible. Therefore, a natural question to ask is who would choose ex ante commit to not selling the loan. There are two countervailing effects. First, in the presence of a CDS market, a loan that does not have an assignment clause may be monitored less than a loan in which there is. This is because the originating bank has the option of purchasing a CDS if its desire to lay off risk is sufficiently high. In this case, it is no longer exposed to the economic risk of the underlying loan and therefore has no incentive to monitor.

A countervailing effect is that clauses restricting future sales are frequently not binding. Currently, under Article 9 of the Uniform Commercial Code, a bank may sell participation in a loan even though the underlying loan agreement has an anti-assignment clause. However, in the presence of an anti-assignment clause, the bank is not allowed to transfer collection rights (effectively monitoring rights) to the buyer. In sum, under current law, such clauses do not actually prevent a bank from selling a loan, but do prevent a bank from transferring the business relationship.\footnote{A detailed description of this Article appears in Schwartz(1999).} In the context of our model, this suggests that a bank with a loan that has an anti-assignment clause has effectively committed that the purchaser will not monitor. Economically, then, an assignability clause is equivalent to a CDS in our model.

3 Characterization of Equilibria

Ownership matters in this framework because the holder of the loan can decrease the default probability. Of course, in some cases the default probability is sufficiently low that monitoring is too costly relative to the benefit. The benefit to monitoring depends both on the increase in success probability and the payoff to the bank if the project succeeds.

The originating bank has superior information about the value of the loan. A bank that knows that the success probability is $p + \Delta$ will never monitor, as monitoring has no effect. However, if the success probability is $p$, the originating bank that retains the loan monitors if

$$C + (p + \Delta)(R^\ell - C) - b > C + p(R^\ell - C).$$

No monitoring

1 A detailed description of this Article appears in Schwartz(1999).
or if \((R^\ell - C) > \frac{b}{\Delta}\).

Clearly, if the credit risk or uncertain payoff, \(R^\ell - C\), is sufficiently small, so that \(R^\ell - C \leq \frac{b}{\Delta}\) no bank would ever monitor and control rights are immaterial, however, for \(R^\ell - C > \frac{b}{\Delta}\), monitoring can add value. In what follows, we restrict attention to cases where the credit exposure of the bank is sufficiently large so that \(R^\ell - C > \frac{b}{\Delta}\).

It is immediate that CDS remove the incentive for a bank to monitor the loan. Effectively, as the bank bears the private cost of monitoring, \(b\), and the benefits accrue to the seller of protection it can never be optimal to monitor.

**Lemma 1**  
(i) An originating bank that buys CDS will never monitor.  
(ii) A bank that buys a loan will monitor it if \(R^\ell - C > \frac{b}{\Delta(1 - \phi)}\).

The second part of the lemma arises because a bank that purchases a loan is less informed that the originator. However, a purchaser updates its beliefs about the performance of the loan, conditional on it being sold. Notice that \(\frac{b}{\Delta(1 - \phi)} \geq \frac{b}{\Delta}\). Therefore, unless there is perfect communication of the originating bank’s private information, there will be a range of credit exposures for which an originating bank would monitor if it retained ownership, but the purchaser of the loan would not. Information in this economy is useful because it allows agents to make an optimal monitoring decision.

The action that the buyer of a loan takes affects the market price of the loan. This is because the originating bank sells to a competitive market, the members of which compete away all their rents.

**Lemma 2**  
An originating bank can lay off credit risk  
(i) Through a loan sale at price \(C + (p + \Delta)(R^\ell - C) - b\) if the purchasing bank plans to monitor.  
(ii) Through a loan sale at price \(C + (p + \theta^o\Delta)(R^\ell - C)\) if the purchasing bank does not plan to monitor.  
(iii) By entering into a CDS for a payment of \((1 - p - \theta^m\Delta)(R^\ell - C)\).

It is clear from Lemma 2 that if a bank wants to lay off credit risk, the decision whether to use loan sales or CDS depends on two things: first, the inference that the market draws from the credit risk transfer method and second, the action that the loan purchaser takes (i.e., whether it plans to monitor or not).

It is immediate that, holding beliefs fixed about the bank types that use the two methods, if the purchasing bank does not plan to monitor, then there is no difference between CDS and loan sales. However, if the beliefs vary so that, for example, one type of credit risk
transfer is associated with high default vehicles, then that method will the eschewed and
the other will dominate. That is, there will be no trade in the dominated vehicle.

By contrast, if the purchasing bank does plan to monitor, then holding beliefs fixed,
loan sales are preferred by some banks to CDS. They are strictly preferred if the originating
bank knows that the loan should be monitored (i.e., the success probability is \( p \)).

**Lemma 3** (i) Suppose that \( \Delta(1 - \phi^o) \leq \frac{b}{R^\ell - C} \), so that a bank who buys a loan will never
monitor, then laying off risk by loan sales or CDS are equivalent.

(ii) Suppose that \( \Delta(1 - \phi^o) > \frac{b}{R^\ell - C} \), so that a purchasing bank will always monitor, then
the originating bank strictly prefers Loan Sales over CDS if \( R^\ell - C > \frac{b}{\Delta(1 - \phi^m)} \).

We now characterize equilibrium beliefs. First, observe that a bank that knows that the
loan is a good one, and has no shock will never sell the loan. The bank cannot credibly
communicate the value of the loan to the market, and has no need to sell its loan. Therefore
in all the equilibria we outline below, \((p + \Delta, 0)\) always retains the loan which he originated.

Let \( \alpha = \frac{\alpha}{1 - \theta} \).

**Proposition 1** There are three types of equilibria:

(i) Complete pooling: If \( \frac{b}{\Delta} < R^\ell - C < \frac{b}{\Delta}(1 + \alpha) \), then all banks except \((p + \Delta, 0)\) shed credit
risk. If there is trade in both the CDS and Loan Sales market then \( p^m = p^o = p + \Delta \frac{\alpha}{\alpha + 1} \).

(ii) Partial Pooling: If \( R^\ell - C > \max[\frac{b}{\beta}, \frac{b}{\Delta(1 - \theta)}] \) then \((p + \Delta, \beta)\) and \((p, \beta)\) sell loans, and
there is no trade in the CDS market. The market’s beliefs are \( p^o = p + \theta \Delta \) and \( p^m = p \).

(iii) No Pooling: If \( \frac{b}{\Delta} < R^\ell - C \leq \frac{b}{\beta} \), only \((p, \beta)\) sheds credit risk through loan sales and
\( p^m = p^o = p \).

The three types of equilibria differ by the inferences of the market about the value of
the project conditional on the bank laying off risk. Notice that for \( \frac{b}{\Delta} < R^\ell - C < \frac{b}{\Delta}(1 + \alpha) \),
more than one equilibrium is sustained for each parameter range.

Proposition 1 allows us to recast Lemma 2 in equilibrium terms. We are able to associate
the level of prices with the uncertainty associated with the eventual outcome.

**Proposition 2** If there is complete pooling and trade in both the CDS and loan sale market,
then the price of a loan is \( [p + \Delta \frac{\alpha}{\alpha + 1}](R^\ell - C) \), else the price of a loan is \( (p + \Delta)(R^\ell - C) - b \).

An immediate implication of Proposition 2 is that if there is trade in both the CDS and
loan sales market for the same name, then there will be a higher variance associated with
the ex post outcome. That is, in a pooling equilibrium all all market data cannot distinguish
between the loan types. By contrast, if there are only loan sales in a particular instrument,
then there is either partial pooling or no pooling. In this case, market data provides a more precise estimate of the future performance of the loan, in that more of the originating bank’s information is impounded into price. Therefore, conditional on a transaction and on transaction data, the variance of outcomes should be smaller. In sum, the presence of the CDS market makes prices in the loan sale market less informative rather than more.

4 The Value of Reputation and Equilibrium Selection

There are two sources of inefficiency in this market. First, if the price of loans is sufficiently low then a firm with a good outside opportunity may not liquidate its position. That is, it may inefficiently retain risk on its balance sheet. Second, if there is pooling, there may be inefficient monitoring as the seller could have received information that the value of the loan was \( p + \Delta \) but cannot communicate this to the buyer. In our simple framework, risk transfer is efficient if banks with a \( \beta \) shock lay off their credit risk. Monitoring is efficient if projects that the originating bank has identified as a high default probability, \( p \), are monitored. Thus, the ex ante value of a loan if there is efficient monitoring and efficient risk transfer is

\[
\pi^* = (p + \Delta)(R^e - C) - (1 - \theta)b + \gamma \beta (R^e - C). \]

The ex ante payoff to each of the three equilibria types can be expressed in terms of the efficient payoff. Let \( \pi^i, i \in \{n, p, c\} \) denote the expected payoff to a loan if there is no pooling, partial pooling and complete pooling. Then,

**Proposition 3** The ex ante payoff to a loan is:

\[
\begin{align*}
\pi^c &= \pi^* + (1 - \theta)b - (1 - \Delta \theta)(R^e - C) \quad \text{if there is complete pooling} \\
\pi^p &= \pi^* - b \gamma \theta \quad \text{if there is partial pooling} \\
\pi^n &= \pi^* - \gamma \theta \beta (R^e - C) \quad \text{if there is no pooling.}
\end{align*}
\]

Clearly, for each equilibrium type there is a different tradeoff between monitoring and risk sharing. The relative payoffs are illustrate in Figure 2.
Figure 2: Ex ante profits to a bank from each of the equilibria

Corollary 1 If equilibrium in the credit risk transfer market exhibits
(i) Complete Pooling then aggregate monitoring is too low, but risk transfer is efficient.
(ii) Partial Pooling then risk transfer is efficient, but aggregate monitoring is too high.
(iii) No Pooling then risk transfer is inefficient but aggregate monitoring is efficient.

Complete pooling leads to efficient risk transfer because all firms (barring (p + Δ, 0)) sell their loans. However, no firms monitor the loans. Therefore, in aggregate, default probabilities are too high.
Corollary 2 If both the CDS and loan sales market are active then the default probabilities are too high relative to the social optimum.

By contrast, in case (ii) of Corollary 1 above, if there is partial pooling all banks that receive shocks lay off their risk. However, the purchaser of the loan cannot distinguish between the banks who know that monitoring is efficient and those that know that monitoring is inefficient and therefore monitor all loans that they buy. In this way, there is a loss of the private monitoring effect $b$ if it is exerted on behalf of the loan that already has a low default probability. Thus, there is too much monitoring. Finally, in case (iii) of Corollary 1 if there is pooling then risk transfer is inefficient because banks with good information about the loan do not lay off the credit risk.

Banks have an incentive to pick the equilibrium that minimizes the ex ante probability of inefficiencies associated with either inefficient monitoring or inefficient risk sharing. In as much as reputation or market convention allows them to select certain equilibria, we characterize parameter ranges under which different market mechanisms for laying off risk are preferred. Using Proposition 1 and Proposition 3 we can characterize when different parameter ranges for which banks ex ante prefer differ credit risk transfer vehicles.

Proposition 4 If

(i) $\frac{b}{\Delta} < R^\ell - C \leq \max \left[ b \left( 1 - \frac{\theta}{1 - \Delta \theta} \right), \frac{b}{\Delta} \right]$ then banks prefer complete pooling

(ii) $\max \left[ b \left( 1 - \frac{\theta + \gamma \theta}{1 - \Delta \theta} \right), \frac{b}{\Delta} \right] \leq R^\ell - C \leq \max \left[ b \left( 1 - \frac{\theta + \gamma \theta}{1 - \Delta \theta} \right), \frac{b}{\beta} \right]$ then banks prefer no pooling,

(iii) $R^\ell - C > \max \left[ b \left( 1 - \frac{\theta + \gamma \theta}{1 - \Delta \theta} \right), \frac{b}{\beta} \right]$ then banks prefer partial pooling.

Proposition 4 establishes that for different yield spreads, banks prefer different equilibria. We note that the yield spread does not translate directly into a default probability because banks monitor to improve the chance of the loan succeeding.

5 Conclusion

We have presented a simple model in which banks choose between laying off risk synthetically through credit default swaps or by selling risky loans. How banks value each of these credit risk transfer methods depends on
6 Proofs

Proof of Lemma 1

(i) If a bank buys CDS then it receives
\[ R^\ell - C \] if default
\[ 0 \] otherwise.

Therefore, the bank’s payoff is \( R^\ell \), independent of the default probability. If the bank monitors, it still receives \( R^\ell \) but also incurs a cost \( b > 0 \).

(ii) If the bank does not have protection it monitors if
\[ C + (R^\ell - C)(p + \Delta) \geq b + C + (p + \phi^o \Delta)(R^\ell - C) \]
The result follows

Proof of Lemma 2

The value of a loan conditional on monitoring is \((p + \Delta)(R^\ell - C)\). The result follows.

Proof of Proposition 1

(i) Complete Pooling

There are two cases: first, if the purchaser of the loan monitors and second if it does not. From lemma 1, if \( R^\ell - C \leq \frac{b}{\Delta(1-\phi^o)} \), then the bank purchasing the loan will not monitor it, however, if the originating bank retains the loan then it monitors.

(a) Suppose that \( R^\ell - C \leq \frac{b}{\Delta(1-\phi^o)} \).

1. Type \((p + \Delta, \beta)\) sells the loan.
\[
C + (p + \Delta)(R^\ell - C) < C + (p + \phi^o \Delta)(R^\ell - C) + \beta(R^\ell - C)
\]
\[
\implies \Delta(1 - \phi^o) < \beta
\]

2. Type \((p, \beta)\) sells the loan rather than keeping it and monitoring if
\[
C + (p + \Delta)(R^\ell - C) - b < C + (p + \phi^o \Delta)(R^\ell - C) + \beta(R^\ell - C)
\]
\[
\implies \Delta(1 - \phi^o) < \frac{b}{R^\ell - C} + \beta
\]
3. Type $(p, 0)$ prefers to sell the loans rather than keeping and monitoring if

\[ C + (p + \phi^o \Delta)(R^\ell - C) > C + (p + \Delta)(R^\ell - C) - b \]

\[ \implies \frac{b}{R^\ell - C} > \Delta (1 - \phi^o). \]

Thus,

\[ p_3^o = p_3^m = p + \Delta \frac{\gamma \theta}{\gamma \theta + (1 - \theta)} \]

\[ (1 - \theta^o) \Delta = \Delta \left[ \frac{1 - \theta}{\gamma \theta + 1 - \theta} \right] \]

Therefore, the conditions become:

\[ R^\ell - C < \frac{b(\gamma \theta + 1 - \theta)}{(1 - \theta) \Delta} \]

\[ \frac{1 - \theta}{\gamma \theta + 1 - \theta} \Delta < \beta \]

(b) Now suppose that the purchaser of the loan monitors. That is, $R^\ell - C > \frac{b}{\Delta(1 - \phi^m)}$. We know from Lemma ?? that a bank prefers to lay off risk with CDS if $(1 - \phi^m) \Delta > \frac{b}{R^\ell - C}$.

1. Type $(p + \Delta, \beta)$ prefers CDS to retaining the loan

\[ C + (p + \phi^m \Delta)(R^\ell - C) + \beta(R^\ell - C) > C + (p + \Delta)(R^\ell - C) \]

\[ \implies \beta > (1 - \phi^m) \Delta \]

2. Type $(p, 0)$ prefers CDS to retaining the loan

\[ C + (p + \phi^m \Delta)(R^\ell - C) > C + (p + \Delta)(R^\ell - C) - b \]

\[ \implies R^\ell - C < \frac{b}{\Delta(1 - \phi^m)} \]

3. Type $(p, \beta)$ prefers CDS to retaining the loan

\[ C + (p + \phi^m \Delta)(R^\ell - C) + \beta(R^\ell - C) > C + (p + \Delta)(R^\ell - C) - b \]

\[ \implies R^\ell - C < \frac{b}{(1 - \phi^m) \Delta - \beta} \]
If
\[ p_3^m = p + \Delta \frac{\gamma \theta}{\gamma \theta + (1 - \theta)} \]

The conditions become:
\[ \implies \left( \frac{1 - \theta}{\gamma \theta + 1 - \theta} \right) \Delta < \beta \]
\[ R^\ell - C < \frac{b}{\Delta} \left( \frac{\gamma \theta + 1 - \theta}{1 - \theta} \right) \]

Recall, this is under the assumption that \( R^\ell - C \geq \frac{b}{(1 - \phi^o)} \Delta \). \( \phi^o = 0 \) will suffice, so \( p^o = p \).

Suppose now, that \( \beta < \Delta (1 - \theta^m) \), then the types who are willing to trade CDS are \((p, \beta)\) and \((p, 0)\) therefore:
\[ p_3^m = p \]
\[ \implies \theta^m = 0 \]

However, this implies that \( \Delta \leq \frac{b}{R^\ell - C} \) which contradicts the maintained assumptions.

(ii) Partial Pooling

(a) First suppose that \( R^\ell - C \leq \frac{b}{\Delta (1 - \phi^o)} \), so that the purchaser will not monitor.

1. Type \((p + \Delta, \beta)\) sells the loan.
   \[ C + (p + \Delta)(R^\ell - C) < C + (p + \phi^o \Delta)(R^\ell - C) + \beta(R^\ell - C) \]
   \[ \implies \Delta (1 - \phi^o) < \beta \]

2. Type \((p, \beta)\) sells the loan rather than keeping it and monitoring if
   \[ C + (p + \Delta)(R^\ell - C) - b < C + (p + \phi^o \Delta)(R^\ell - C) + \beta(R^\ell - C) \]
   \[ \implies \Delta (1 - \phi^o) < \frac{b}{R^\ell - C} + \beta \]

3. Type \((p, 0)\) prefers to keep the loan and monitor if
   \[ C + (p + \phi^o \Delta)(R^\ell - C) \leq C + (p + \Delta)(R^\ell - C) - b \]
   \[ \implies \frac{b}{R^\ell - C} \leq \Delta (1 - \phi^o). \]
Thus, there is partial pooling if

\[ R^\ell - C = \frac{b}{\Delta(1 - \phi)} \]

\[ \Delta(1 - \phi) < \beta \]

(b) Now suppose that \( R^\ell - C > \frac{b}{(1 - \phi)\Delta} \), so that loan buyers will monitor the loan. If \((1 - \phi_m)\Delta > \frac{b}{R^\ell - C}\), then banks prefer loan sales to CDS.

1. Type \((p + \Delta, \beta)\) prefers to sell the loan:

\[ C + (p + \Delta)(R^\ell - C) - b + \beta(R^\ell - C) \geq C + (p + \Delta)(R^\ell - C) \]

\[ \implies \beta \geq \frac{b}{R^\ell - C} \]

2. Type \((p, \beta)\) always sells the loan

3. Type \((p, 0)\) prefers not to sell the loan because:

\[ C + (p + \Delta)(R^\ell - C) - b = C + (p + \Delta)(R^\ell - C) - b \]

If \(\beta > \frac{b}{R^\ell - C}\), then \((p + \Delta, \beta)\) and \((p, \beta)\) sell. So loan sales are uninformative and

\[ p^0_3 = p + \Delta \theta \]
\[ p^m_3 = p \]
\[ R^\ell - C > \max \left[ \frac{b}{\beta}, \frac{b}{1 - \phi} \Delta \right] \]

(iii) No Pooling,

Now suppose that \( R^\ell - C > \frac{b}{(1 - \phi)\Delta} \), so that loan buyers will monitor the loan. If \((1 - \phi_m)\Delta > \frac{b}{R^\ell - C}\), then banks prefer loan sales to CDS.

1. Type \((p + \Delta, \beta)\) prefers to keep the loan:

\[ C + (p + \Delta)(R^\ell - C) - b + \beta(R^\ell - C) < C + (p + \Delta)(R^\ell - C) \]

\[ \implies \beta < \frac{b}{R^\ell - C} \]
2. Type \((p, \beta)\) always sells the loan

3. Type \((p, 0)\) prefers not to sell the loan because:

\[
C + (p + \Delta)(R^\ell - C) - b = C + (p + \Delta)(R^\ell - C) - b
\]

So, for \(\beta \leq \frac{b}{R^\ell - C}\), the only loans that are sold are those in which the project succeeds with probability \(p\). Thus,

\[
p_3^o = p \\
p_3^m = p
\]

**Proof of Proposition 3**

(i) Complete Pooling:

\[
(p + \Delta, \beta) \implies C + (p + \Delta\frac{\gamma \theta}{\gamma \theta + 1 - \theta})(R^\ell - C) + \beta(R^\ell - C) \\
(p + \Delta, 0) \implies C + (p + \Delta)(R^\ell - C) \\
(p, \beta) \implies C + (p + \Delta\frac{\gamma \theta}{\gamma \theta + 1 - \theta})(R^\ell - C) + \beta(R^\ell - C) \\
(p, 0) \implies C + (p + \Delta\frac{\gamma \theta}{\gamma \theta + 1 - \theta})(R^\ell - C)
\]

The expected value of the contract:

\[
C + (p + \theta \Delta)(R^\ell - C) + \gamma \beta(R^\ell - C)
\]

Also,

\[
\frac{b}{\Delta} < R^\ell - C < \frac{b(\gamma \theta + (1 - \theta))}{(1 - \theta)\Delta} \text{ and } \frac{1 - \theta}{\theta + 1 - \theta} < \beta
\]
\[(p + \Delta, \beta) \implies C + (p + \Delta \frac{\gamma \theta}{\gamma \theta + 1 - \theta})(R^e - C) + \beta(R^e - C)I\]
\[(p + \Delta, 0) \implies C + (p + \Delta)(R^e - C)\]
\[(p, \beta) \implies C + (p + \Delta \frac{\gamma \theta}{\gamma \theta + 1 - \theta})(R^e - C) + \beta(R^e - C)\]
\[(p, 0) \implies (p + \Delta \frac{\gamma \theta}{\gamma \theta + 1 - \theta})(R^e - C)\]

In this case, the expected value of the contract is

\[C + (R^e - C)(\theta \Delta + p) + \gamma \beta(R^e - C)\]

(ii) Partial Pooling

Recall, the price of the loan is \((p + \Delta)(R^e - C) - bI\).

\[(p + \Delta, \beta) \implies C + (p + \Delta)(R^e - C) - b + \beta(R^e - C)\]
\[(p + \Delta, 0) \implies C + (p + \Delta)(R^e - C)\]
\[(p, \beta) \implies C + (p + \Delta)(R^e - C) - b + \beta(R^e - C)\]
\[(p, 0) \implies C + (p + \Delta)(R^e - C) - b\]

In this case the expected value of the contract is:

\[C + (p + \Delta)(R^e - C) - b(\gamma \theta + 1 - \theta) + \gamma \beta(R^e - C)\]

(iii) No pooling

\[R^e - C > \frac{b}{\Delta}, \text{ and } \beta < \frac{b}{R^e - C}\]

\[(p + \Delta, \beta) \implies C + (p + \Delta)(R^e - C)\]
\[(p + \Delta, 0) \implies C + (p + \Delta)(R^e - C)\]
\[(p, \beta) \implies C + (p + \Delta)(R^e - C) - b + \beta(R^e - C)\]
\[(p, 0) \implies C + (p + \Delta)(R^e - C) - b\]
Hence, the Expected value of the contract is:

\[ C + (p + \Delta)(R^l - C) - (1 - \theta)b + \gamma(1 - \theta)\beta(R^l - C) \]

\[ \square \]

**Proof of Proposition 4** We first establish pair-wise conditions on the different possible ex ante profits to determine when a bank would be prefer one equilibrium over another.

(i) \( \pi^n \) is weakly preferred to \( \pi^p \) if

\[ \pi^* - \gamma \theta \beta(R^l - C) \geq \pi^* - b \gamma \theta \]

\[ \implies \frac{b}{\beta} \geq R^l - C \]

(ii) \( \pi^c \) is weakly preferred to \( \pi^p \) if

\[ \pi^* + (1 - \theta)b - (1 - \Delta \theta)(R^l - C) \geq \pi^* - b \gamma \theta \]

\[ \implies b \left( \frac{1 - \theta + \gamma \theta}{1 - \Delta \theta} \right) \geq R^l - C \]

(iii) \( \pi^c \) is weakly preferred to \( \pi^n \) if

\[ \pi^* + (1 - \theta)b - (1 - \theta \Delta)(R^l - C) \geq \pi^* - \gamma \theta \beta(R^l - C) \]

\[ \implies \frac{(1 - \theta)b}{1 - \Delta \theta + \gamma \theta \beta} \geq R^l - C \]

We also observe that:

\[ b \left( \frac{1 - \theta + \gamma \theta}{1 - \Delta \theta} \right) \geq \frac{b}{\Delta} \]

\[ \implies \Delta(1 + \gamma \theta) \geq 1. \]

**Case 1**

Suppose that \( \Delta(1 + \gamma \theta) \geq 1 \), so that \( b \left( \frac{1 - \theta + \gamma \theta}{1 - \Delta \theta} \right) \geq \frac{b}{\Delta} \). Then

(a) If \( \frac{b}{\Delta} \leq \frac{b}{\beta} \), then for all feasible parameters \( \pi^p \) is preferred to \( \pi^n \). So, for \( \frac{b}{\Delta} \leq R^l - C \leq b \left( \frac{1 - \theta + \gamma \theta}{1 - \Delta \theta} \right) \) the banks prefer \( \pi^c \) whereas for \( R^l - C > b \left( \frac{1 - \theta + \gamma \theta}{1 - \Delta \theta} \right) \) they prefer \( \pi^p \).

(b) Suppose that \( \frac{b}{\Delta} < \frac{b}{\beta} \), then for \( \frac{b}{\Delta} < R^l - C \leq \frac{b}{\beta} \), the banks prefer \( \pi^n \), whereas for \( R^l - C > \frac{b}{\beta} \) the banks prefer \( \pi^p \). Thus,

(i) If \( b \left( \frac{1 - \theta + \gamma \theta}{1 - \Delta \theta} \right) < \frac{b}{\beta} \), then for \( \frac{b}{\Delta} < R^l - C < b \left( \frac{1 - \theta + \gamma \theta}{1 - \Delta \theta} \right) \) banks prefer \( \pi^c \), and for \( b \left( \frac{1 - \theta + \gamma \theta}{1 - \Delta \theta} \right) \leq R^l - C < \frac{b}{\beta} \) banks prefer \( \pi^n \), while for \( R^l - C \geq \frac{b}{\beta} \) banks prefer \( \pi^p \).

(ii) If \( b \left( \frac{1 - \theta + \gamma \theta}{1 - \Delta \theta} \right) \geq \frac{b}{\beta} \), then for \( \frac{b}{\Delta} < R^l - C \leq \frac{b}{\beta} \) banks prefer \( \pi^c \) and for \( \frac{b}{\beta} < R^l - C < b \left( \frac{1 - \theta + \gamma \theta}{1 - \Delta \theta} \right) \) banks prefer \( \pi^c \), and for \( R^l - C > b \left( \frac{1 - \theta + \gamma \theta}{1 - \Delta \theta} \right) \) banks prefer \( \pi^p \).
Case 2 Suppose that $\Delta(1 + \gamma \theta) < 1$, so that $b \left( \frac{1 - \theta + \gamma \theta}{1 - \Delta \theta} \right) < \frac{b}{\Delta}$. Then for $\frac{b}{\Delta} < R^\ell - C \leq \frac{b}{\beta}$ banks prefer $\pi^n$, whereas for $R^\ell - C > \frac{b}{\beta}$, banks prefer $\pi^p$.\[\Box\]
7 Regulatory Constraints

Suppose that the bank faces a risk limit, in that the ex ante variance (beliefs are relative to the public information) of its position is restricted. Specifically, government regulators require that the variance of the bank’s payoffs are less than $\bar{V}$. The variance of the original project is:

$$(R^\ell - C)^2 I^2 p(1 - p) \leq \bar{V}$$

Another project that is perfectly correlated would violate the bank’s variance constraint: In particular,

$$4(R^\ell - C)^2 I^2 p(1 - p) > \bar{V}$$

Without modelling specific details, a bank would then

1. Raise extra equity capital. This would increase $\bar{V}$. If this is costly, then a reduced form way of modelling it, would be to have a private value for risk reduction.

2. In a full blown model, the bank could also take a smaller portion of the new loan. However, we are implicitly assuming that this is not possible.

3. Because the bank retains all the surplus at the contracting stage, the friction associated with the new loan is not reflected in the contract terms of the new loan.
References


