Predatory Patent Litigation *

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Abstract

Despite their expertise in patent law, the most litigious patent assertion entities (PAEs) frequently file dubious infringement claims on which they are ostensibly very unlikely to turn a profit. Thus one might conjecture that these PAEs are mistaken to follow through on their litigation threats when their chances of coming out ahead are so scant. To the contrary, this paper demonstrates that this is in fact a calculated strategy of *predatory patent litigation*: by following through on its threats of seemingly irrational litigation, the PAE convinces other producers that these threats are credible, leading them to accept licensing offers they would ordinarily rebuff. This allows the PAE to garner substantial licensing revenues using low quality patents that would otherwise be difficult or impossible to monetize. Like predatory pricing, this strategy involves a short run loss that is recouped over time through supra-competitive pricing.

This paper develops a stylized dynamic model of patent assertion and reputation building by a PAE with low quality patents. The model has a unique equilibrium that involves predatory patent litigation, and in which the PAE intermittently forfeits and rebuilds its litigious reputation over time. Predatory patent litigation generates substantial social costs, and creates a perverse incentive for patent applicants to seek coverage of technologies so obvious or non-novel that they are likely to be widely unintentionally infringed by unsuspecting producers. Importantly, fee shifting will not solve the problem. Rather, it will lead predatory PAEs to focus their ire on small, vulnerable targets, such as technology startups, for whom litigation may be crippling even if attorneys fees are ultimately recouped. Potential defendants could better deter predatory PAEs by entering a *litigation cost-sharing agreement* in which members jointly pay one anothers litigation costs and litigate all meritless claims to judgment. If properly limited in scope, such an arrangement will not materially undermine meritorious infringement actions.

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1 Introduction

Patent assertion entities (PAEs) – pejoratively known as "patent trolls" – are firms whose business operations consist primarily in patent assertion, licensing and litigation. Such firms typically do not produce anything covered by their patents, and are therefore frequently referred to as "non-practicing entities." Some of the most active and litigious PAEs are the ones with the weakest patents, which are often excessively broad and likely invalid.[1] These PAEs frequently file infringement suits on which they ostensibly have little or no chance of turning a profit. But in fact this strategy is not at all mistaken or foolhardy. Rather, it is part of a calculated reputation building strategy of *predatory patent litigation* under which a PAE follows through on its seemingly hopeless litigation threats in order to compel future defendants to pay more lucrative settlements. This allows the PAE to monetize bad patents that would otherwise be difficult or impossible to license.

The social desirability of PAEs has been hotly debated in recent years.[2] Proponents of these firms contend that they enhance welfare by providing a vehicle for small inventors to monetize their ideas, and by improving patent market liquidity.[3] By contrast, opponents argue that PAEs inflate social costs[4], and inhibit innovation[5] by creating a more contentious competitive environment for inventors, subjecting them to licensing "shakedowns" that erode the profitability of their new ideas.

Despite being very experienced in patent litigation, the most litigious PAEs perform relatively poorly in court. Allison, Lemley & Walker (2011) show that in cases involving the most litigated patents (those litigated 8 or more times), NPEs win less than 10 percent of their cases when litigated to judgment. Further, these suits make up a large majority of PAE litigation. Chien (2012) finds that in PAE litigation occurring during 2011-2012, 61 percent of defendants were sued by PAEs who had litigated on the same patents 8 or more times. Moreover, recent empirical evidence suggests that, even when PAEs win, their damages tend to be slightly smaller than those of practicing entities.[6] And, as illustrated by some PAE lawsuits discussed below, there are many examples of situations in which PAEs have initiated lawsuits based on alleged infringement of patents so overreaching in scope that they are ostensibly certain to be held invalid if the case reaches final judgment.

In light of this, it might appear that these litigious PAEs are often mistaken to pursue litigation so fervently. As Allison, Lemley and Walker (2011) write, "it appears that NPEs are not as worried about losing as they should be." However, this paper argues that many of the most litigious PAEs' are in fact engaging in a strategy of predatory patent litigation, and that this is actually the most effective way to monetize bad patents. The strategy plays out as follows: the PAE asserts a low

[1] A patent, or more specifically a patent claim, is declared invalid by a court if the subject matter claimed by the patent is held to be ineligible for patent protection.

[2] See Hagiu & Yoffie (2012) or Lemley & Melamed (2013) for background information on the subject.

[3] See, e.g. McDonough (2006).

[4] See Bessen, Meurer & Ford (2011).

[5] See Scott Morton & Shapiro (2013).

[6] See Mazzeo, Hillel & Zyontz (2013).

quality patent against a firm alleged to have unintentionally infringed the patent. Typically the patent is arguably infringed, but very likely invalid. The PAE threatens to sue if its licensing demands are not met. However, defendants recognize that the PAE is very likely to lose money if it litigates, and so some of them assume the threat is most likely a bluff, leading them to reject the PAE's offer. But rather than giving up, the PAE aggressively litigates, despite expecting to lose money on the suit. This litigation imposes significant costs on the defendant, regardless of the outcome or whether a settlement is reached. All of this is observed by other potential defendants who subsequently view the PAEs threats as more credible than they had previously thought, making them more amenable to the PAE's licensing terms. Importantly, a PAE need not take its predatory suits all the way to judgment in order for predatory patent litigation to be successful. As long as litigation proceeds far enough to impose substantial costs on the defendant, the PAE will have made its point.

One good example of predatory patent litigation involves a PAE named Innovatio IP Ventures.[7] In 2011, Innovatio purchased a number of patents from Broadcom Corporation. Claiming that these patents covered the provision of wifi internet access, Innovatio began asserting its patents against a large number of small businesses – primarily coffee shops, restaurants and grocers – who offered wifi access to customers. Its licensing demands are small, typically between $2000 and $5000. Of course, given the ubiquity of wifi access, if this claim were meritorious then the patents' value would be nothing short of astonishing. But Innovatio's modest settlement terms indicate that it probably did not expect to win money in court. And yet, despite the likelihood of losing money on litigation, Innovatio filed many lawsuits against businesses that rejected its licensing demands. Eventually, wireless router manufacturer Cisco stepped in, offering Innovatio a multi-million dollar settlement to stop filing suits against customers using its routers to provide wifi internet access.[8]

Technology markets are widely regarded as having an acute problem with low quality patents. In a USPTO interview, then-Vice-President of Oracle Corporation Jerry Baker stated that "[Oracle's] engineers and patent counsel have advised me that it may be virtually impossible to develop a complicated software product today without infringing numerous broad existing patents."[9] Consequently technology markets offer many opportunities for predatory patent litigation. For example, in 2013 a PAE called Lumen View Technology demanded a license fee from a technology startup for using a matching process on its website that served to match customers with products and sellers – a fairly simplistic process allegedly covered by Lumen View's patent, which described the process as a "System and Method for Facilitating Bilateral and Multilateral Decision-Making."

Lumen View asserted its patent via a letter stating that the defendant should "be advised that [Lumen View] is prepared for full scale litigation to protect its rights," and even threatening to

[7] See http://www.techdirt.com/blog/wireless/articles/20111001/00365416161/patent-troll-says-anyone-using-wifi-infringes-wont-sue-individuals-this-stage.shtml or http://essentialpatentblog.com/2013/01/catching-up-on-innovation-ip-ventures-llcs-litigation-activities/

[8] The settlement vaule was approximately $2.7 million. See Joe Mullin, "Wi-Fi patent troll will only get 3.2 cents per router from Cisco". Ars Technica, February 6, 2014. Available at http://arstechnica.com/tech-policy/2014/02/cisco-strikes-deal-to-pay-wi-fi-patent-troll-3-2-cents-per-router/

[9] United States Patent and Trademark Office, "Public Hearing on Use of the Patent System to Protect Software-Related Inventions," January 26-27, 1994.

increase its licensing demands if the defendant did not immediately accept.[10] When the defendant refused to pay, Lumen View aggressively litigated, despite the virtual certainty of losing. The district court ultimately held that the patent was blatantly invalid, remarking that "[t]here is no inventive idea here" and that the patented matchmaking process was " a fundamental process that has occurred all through human history."[11] Despite the loss, Lumen View remains an active PAE.

As the Lumen View case illustrates, predatory PAEs are often not dissuaded by the possibility that their patents will be held invalid. This is in part because they can generally acquire more suitable patents from operating companies – a practice known a "patent privateering." It is also a consequence of the fact that, because PAEs generally do not produce anything, they stand to lose less in court. Indeed, in a recent suit filed by a PAE called Eon-Net, the court noted that "while Eon-Net risked licensing revenue should its patents be found invalid or if a court narrowly construed the patents claims to exclude valuable targets, Eon-Net did not face any business risk resulting from the loss of patent protection over a product or process. Its patents protected only settlement receipts, not its own products."[12]

This paper develops a stylized dynamic model of patent assertion, litigation and reputation building by a PAE with low quality patents. The model has a unique Markov perfect equilibrium that generally involves predatory patent litigation along the equilibrium path. The PAE gains a strong reputation for aggressive litigation by following through on a litigation threat despite expecting to lose money on the suit. The equilibrium exhibits interesting dynamics, with the PAE intermittently forfeiting and rebuilding a litigious reputation over time. The framework differs from conventional reputation models in a number of respects, allowing for particularly tractable analysis while still capturing the underlying economic intuition for reputation building. Given that a large majority of patent disputes settle, and that the specific terms of these settlements are rarely disclosed, this theoretical investigation allows us to address questions that would be difficult or impossible to answer empirically.[13]

Predatory patent litigation imposes substantial social costs without doing anything to promote innovation. A technology that is legitimately novel and non-obvious will typically give rise to strong infringement claims, making predatory litigation unnecessary to garner license fees. Further, by its nature, predatory patent litigation does nothing to promulgate new ideas. The practice embodies what one might call a "wait and sue" approach to patent licensing: rather than seeking willing licensees ex ante, the PAE waits until it identifies firms that have unintentionally

[10] A full copy of Lumen View's letter is available on the website TrollingEffects.org at URL https://trollingeffects.org/demand/lumen-view-technology-2013-05-30

[11] See *Lumen View Tech. LLC v. Findthebest.com, Inc.*, 13 CIV. 3599 (DLC), 2013 WL 6164341 (S.D.N.Y. Nov. 22, 2013)

[12] See *Eon-Net LP v. Flagstar Bancorp*, 653 F. 3d 1314, (Fed. Cir. 2011).

[13] Most infringement claims are settled before reaching final judgment, and many patent disputes are resolved before litigation even commences. Furthermore, even if it were possible to determine the terms of a defendant's settlement, there is ostensibly no good way of estimating the merit of the infringement allegation that induced it. The reputation component of predatory patent litigation creates additional problems. Given the unavailability of data on settlement terms, there is apparently no way to observe the positive reputation effects that appear to motivate predatory litigation.

infringed, meaning they independently invented the disputed technologies without realizing they were patent-protected, at which point the PAE steps in and threatens litigation. The profitability of this approach hinges on the propensity for widespread independent invention, which is a signal that patent protection was not warranted in the first place. Indeed, these technologies were going to be widely discovered either way; the patentee just happened to be the first person to stake his flag. In this way, predatory patent litigation will simply encourage applications for excessively broad patents covering technologies so obvious or non-novel that they are likely to be widely unintentionally infringed by unsuspecting firms. This exacerbates the "patent thicket" problem addressed in Shapiro (2001), making it difficult for firms and innovators to implement their own ideas without exposing themselves to some risk of low merit infringement litigation.

Many scholars and practitioners have rallied in favor of an expanded fee shifting rule, which would require plaintiffs in frivolous infringement suits to pay the defendants' attorneys' fees. This would make things easier on defendants in cases that reach judgment, and it would likely lead predatory PAEs to give up in some situations where they would otherwise have litigated, but it is unlikely to provide the broad resolution its proponents are hoping for. The problem is that the injury a defendant-firm suffers as a result of litigation is generally not limited to the amount it spends on attorneys. The need to divert resources to litigation can impose substantial difficulties and opportunity costs on the defendant's business operations. For example, the firm might have to file for bankruptcy, scale back production, or pass on valuable opportunities for company growth. And, as long as a PAE can substantially injure a defendant, he can achieve the desired intimidation effect on future licensing negotiations with other firms. As a consequence, fee shifting will tend to increase a predatory PAE's incentive to target small, vulnerable defendants for whom litigation may be crippling even if attorney's fees are ultimately recouped.

Potential defendants could better shield themselves from predatory patent litigation by forming a *litigation cost-sharing agreement* (LCSA). This is a contractual agreement such that, when a member firm is sued for infringement, his litigation costs are split among all members as they arise, provided that (1) the infringement claim exhibits some contractually specified characteristics aimed at identifying predatory claims, or it is deemed sufficiently unlikely to succeed on the merits by an impartial arbiter; and (2) the defendant agrees not to settle. Under an LCSA, defendants are much less daunted by the prospect of litigation, substantially hindering the PAEs to garner license fees from member firms. Moreover, by prohibiting settlement, an LCSA destroys a predatory PAE's leverage once litigation commences. If an LCSA's provisions are properly limited to predatory infringement claims, such arrangements are unlikely to arouse antitrust scrutiny, and will not deter plaintiffs from filing meritorious claims against member firms.

Importantly, this paper is focused specifically on PAEs who engage in predatory patent litigation. It does not make categorical claims about the universe of all PAE activity. Many PAEs do not engage in predatory patent litigation at all, instead focusing on acquiring high value patents that can be asserted against a few big players. As Lemley and Melamed (2013) write, "these trolls think they have a patent that reads on a significant area of technology, and it is very important to them that their patent is held valid and infringed." Thus, in stark contrast with predatory PAEs, these firms are deeply concerned with making strong infringement claims, which they typically direct at a few big firms rather than a large number of small ones. While a few research papers address PAEs who specialize in asserting low quality patents against a large number of defendants,[14] the vast majority do not distinguish among different PAE licensing strategies. Furthermore, there appear to be no existing studies that address reputation building by PAEs, or which specifically analyze the strategic use of negative expected value litigation by PAEs.

The remainder of the paper is organized as follows: section 2 develops a dynamic model of patent assertion, licensing, litigation and reputation building by a PAE in possession of low quality patents. Section 2.1 extends the model to assess the impact of a fee shifting rule. The technical results from these sections are briefly summarized at the beginning of section 3, which focuses on the impact of predatory patent litigation on innovation and patenting. Section 4 addresses the ability of litigation cost-sharing agreements to deter predatory litigation without undermining the enforceability of good patents. Section 5 concludes.

2 Model

We begin by developing a simple dynamic model of patent assertion and litigation by a PAE who asserts one of its patents in every period. We then extend the game to allow for reputation building through aggressive litigation. Time is discrete and indexed by $t = 0, 1, 2, ...$ Player 1 is a PAE (and a non-practicing entity[15]), and is a long run player who asserts a patent against (and potentially sues) player 2 in every period. Player 2 is a short run player representing a sequence of distinct potential defendants – one in each period – each of whom cares only about the payoff he earns in that period. Patent assertion involves the following interaction: player 1 accuses player 2 of infringement, demands a license fee f and threatens to sue if this demand is not met. Player 2 can accept or reject. If player 2 rejects, player 1 can either follow through on his litigation threat or give up. Note that we interpret the dynamic game as involving assertion of different patents over time, and thus the possibilities of patent expiration or invalidation will not prevent player 1 from continuing to operate.

If player 1 chooses to litigate, then he faces an expected *litigation injury* equal to z, which is independently drawn from support $[\underline{z}, \overline{z}]$ at the beginning of each period according to the distribution Φ. Player 1's litigation injury is simply the negative of his expected litigation payoff (expected damages receipts minus expected costs). We can think of it as reflecting the quality of a particular infringement claim, with more dubious claims corresponding to higher values of z. Intuitively, worse claims will tend to be harder to argue, and will engender smaller expected monetary damages given that they are less likely to win. We assume that $\overline{z} > \max\{\underline{z}, 0\}$, and we allow for (but do not require) the possibility that litigation sometimes has positive expected value, i.e. $\underline{z} < 0$. The distribution Φ is continuous on $[\underline{z}, \overline{z}]$, and strictly increasing and continuously differentiable on $(\underline{z}, \overline{z})$. Φ therefore admits a continuous, positive-valued density, ϕ. The distribution is assumed not to have divergent limiting behavior, so that $\lim_{z \to \underline{z}} \phi(z)$ and $\lim_{z \to \overline{z}} \phi(z)$ exist and are finite. Finally, the realization of z in a particular period t is denoted z_t.

[14] Lemley and Melamed (2013) provide some discussion of these PAEs, which they call "bottom feeders." Consistent with our results, they contend that this typically involves asserting bad patents against small operating companies that are particularly vulnerable to litigation.

[15] A non-practicing entity is a patent holder that does not sell anything covered by its own patents. This term is often used interchangeably with "patent assertion entity."

Player 2 always loses money on litigation. He just happens to lose more when player 1 wins the suit. Intuitively, player 2's litigation injury (litigation costs plus expected damages payments) will tend to be inversely related to player 1's. A more dubious infringement claim will tend to be easier for player 1 to defeat, and is less likely to require him to pay damages. Given a realization z, player 2's litigation is given by $y(z)$, where $y(\cdot)$ is nonincreasing in z with $y(z) > 0$ for all z. We assume that any decision by the court will be limited to a damages award, which is reasonable because the courts very rarely grant injunctions for non-practicing entities.[16] Thus a win by player 1 will simply involve a transfer between the players, which means that litigation is a negative-sum game, given that it is costly regardless of outcome. This implies $z + y(z) > 0$ for all z. Figure 1 depicts the stage game's extended form, with payoffs given in parentheses.

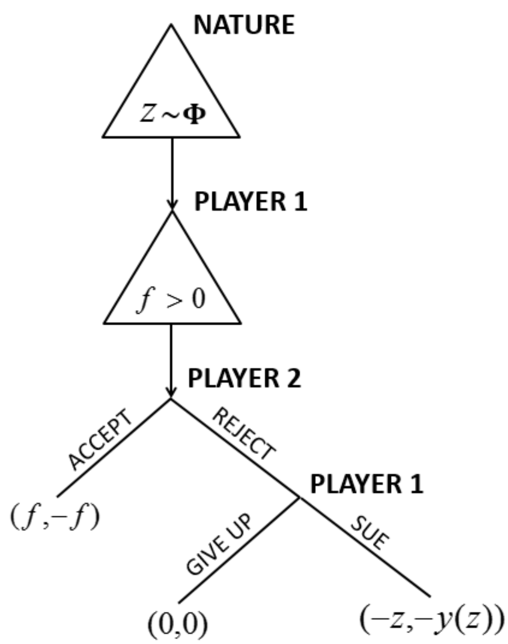

Figure 1: Stage Game

Note that we require player 1 to make a positive licensee fee offer.[17] Clearly litigation is subgame perfect in the stage game only if $z \leq 0$, in which case the stage game's subgame perfect Nash equilibrium (SPNE) involves player 1 offering $f = y(z)$; player 2 accepting any $f \leq y(z)$ and otherwise rejecting; and player 1 always litigating following a rejection. As for the dynamic game, the Folk Theorem tells us that virtually any payoffs may arise in a SPNE. Some SPNEs have the flavor of reputation, but they lack any intuitive explanation for why player 1 should have a tough

[16] The Supreme Court's decision in *eBay v. MercExchange* strengthened the requirements for injunction awards. The revised standard makes it nearly impossible for a non-practicing entity to receive an injunction. See *eBay Inc. v. MercExchange, L.L.C.*, 547 U.S. 388 (2006).

[17] This is because $f = 0$ should be viewed as equivalent to giving up, which player 1 still free to do. This allows us to simplify the algebra without impacting the game's equilibrium (it will be easy to verify that $f = 0$ is a best response only when player 1 does not care what fee he offers.)

reputation in the first place.[18] In their seminal reputation paper, Fudenberg and Levine (1989) resolve this problem by assuming that players place positive probability on a type of the long run player that always chooses to play aggressively, which serves to limit the vast set of equilibria to those in which player 1 earns relatively large payoffs. Consequently, their model merely provides an theoretical explanation for why player 1 might have a tough reputation; its equilibria do not involve actual reputation building, and thus do exhibit any nontrivial equilibrium dynamics.

To allow for reputation effects, we modify the game as follows: player 2 takes on one of two possible types, the realization of which is privately observed by him at the beginning of the period. In particular, in each period player 2 is a "normal type" (the same type he maintains in the unperturbed game) with probability $1-p$, and an "impressionable type" with probability p, where $p \in (0,1)$. An impressionable type is one that is intimidated when player 1's engages in predatory litigation (i.e. litigation with $z > 0$), and emboldened when player 1 gives up. More specifically, this is a behavioral type[19] that focuses on the aggressiveness (or non-aggressiveness) of player 1's recent conduct in periods with $z > 0$ when forming beliefs about how he will behave in such periods in the future. Importantly, when litigation is rational in the static sense ($z \leq 0$), player 1 cannot develop a reputation for predatory litigation, because even the impressionable type understands that litigation is the sensible thing to do under the circumstances.

This model's framework differs from existing reputation models in that it is the short run player and not the repeat-player who can take on one of several possible types. In conventional models, reputation derives from uncertainty about some characteristics of the long run player,, and its sustainability therefore depends on the player's ability to keep his true circumstances private.[20] By contrast, reputation in this model is a little like fiat money. It does not matter that some agents understand a reputation to be artificial or baseless; as long as *some* agents view it as a relevant factor in making decisions, it is rational for *all* agents to view it as such. Accordingly, this take on reputation is less about uncertainty surrounding a particular player, and more about optimal decision making in the presence of some impressionable actors.

Formally, player 1's reputation is an elucidation of the way he is presently viewed by the impressionable type. Unlike conventional reputation models, which involve Bayesian updating over a continuum of beliefs, this framework utilizes a simpler, discretized specification intended to capture the same intuition while enabling more tractable analysis of the issues at hand. Depending on its perception of the plaintiff's litigiousness, the impressionable type attaches

[18] For example, if player 1 is sufficiently patient, the following is a SPNE of the dynamic game: (1) players play stage game SPNE strategies when $z \leq 0$; (2) when $z > 0$, player 1 always offers $f = y(z) - \varepsilon$ for some small $\varepsilon > 0$ and always litigates following a rejection; and (3) when $z > 0$ player 2 accepts if and only if $f \leq y(z) - \varepsilon$, unless player 1 has previously deviated from the strategy in (2), in which case player 2 rejects everything. This feels a little like a reputation scenario, but player 1 is simply endowed with this tough reputation; he does nothing to earn it, and if he loses it he can never get it back.

[19] The inclusion of a behavioral type is not a particularly serious departure from conventional reputation models. In most such models, reputation derives from the fact that there is a positive probability that the long run player is a "tough" type, which is really just a behavioral type that always plays aggressively. The only difference is that, in this model, the behavioral type is certain to appear, albeit intermittently.

[20] See Fudenberg & Levine (1989), which developed a generalized framework to which much of the reputation literature adheres. Some models are more stylized and involve actual reputation dynamics along the equilibrium path. See, e.g., Tadelis (1999).

one of two possible reputations to player 1: strong (S) or weak (W). The reputation with which player 1 enters period t is denoted $R_t \in \{S, W\}$. If player 1 has a strong reputation, then the impressionable type believes he will always follow through on his litigation threat, even if this involves a substantial expected loss. Thus the impressionable type accepts every offer that is weakly preferable to litigation (every $f \leq y(z)$), no matter the value of z. By contrast, if player 1 has a weak reputation, then the impressionable type believes player 1 will litigate only if he expects to turn a profit on the suit ($z \leq 0$), and will otherwise give up. In this case the impressionable rejects every offer when $z > 0$, and accepts any $f \leq y(z)$ when $z \leq 0$.

The process by which player 1's reputation evolves is designed to capture the underlying economic intuition for reputation building, namely that it occurs when someone's conduct surprises a player and leads him to adjust his beliefs about a rival. Thus player 1 enjoys a reputation effect (i.e. a change in reputation) whenever someone's conduct surprises the impressionable type, which happens when either: (*i*) player 1 had a weak reputation and was unexpectedly litigious or unexpectedly intimidated player 2 into licensing a bad patent, or (*ii*) player 1 had a strong reputation and unexpectedly gave up. There are no reputation effects when there are no surprises, and thus player 1's reputation is unchanged in periods with $z \leq 0$. In these periods the impressionable type understands that player 1's litigation (and by extension player 2's acceptance) is perfectly sensible, and he is therefore unsurprised by it.[21] Player 1 could surprise the impressionable type in these periods only by giving up, but such a decision would never be optimal. This results in the following reputation transition process:

- $R_{t+1} = S$ if in period t either: (*i*) $z_t > 0$ and player 1 litigated following a rejection; (*ii*) $z_t > 0$ and player 2 accepted; or (*iii*) $z_t \leq 0$, $R_t = S$ and either player 2 accepted or player 1 litigated following a rejection.
- $R_{t+1} = W$ if in period t either: (*i*) player 1 gave up following a rejection; or (*ii*) $z_t \leq 0$ and $R_t = W$.

Note that this specification sometimes allows player 1 to garner a positive reputation effect simply by convincing player 2 to accept his offer. This differs from the prevailing models of reputation, which allow player 1's reputation to improve only when he takes some unexpected action. In those models, such actions are the only way to surprise short run players and induce them to adjust their beliefs. However, this discrepancy is not problematic; it is merely a result of the fact that this model enables *both* players to generate reputation effects. This is a consequence of the fact that player 2 has multiple types that may disagree on how player 1 is likely to act in a particular situation, which means that player 2 will occasionally be surprised by the actions of one of his predecessors, which may lead him to adjust his beliefs about player 1. In particular, if a normal type of player 2 accepts an offer in a period with $R_t = W$, the impressionable type is surprised. He would have rejected the offer. He consequently assumes that his predecessor knew something he did not know himself, and that player 1 must have been prepared to take some action that he was not anticipating. He thus updates his beliefs concerning player 1 accordingly. By contrast, in models with a single type of player 2, no such reputation effects can arise: player 2

[21] In principal we could allow player 1 to generate a positive reputation effect by demanding an unreasonably high license fee and forcing litigation. Specifically, if $R_t = W$ and $z_t \in (-y, 0]$, we could allow player 1 to gain a strong reputation by offering an unacceptably high offer (any $f > y$ would suffice) in order to compel litigation and make an example of player 2. Given that player 1 could have garnered a larger payoff of y without having to litigate, his decision not subgame perfect in the stage game, just like predatory litigation. However, the author is not aware of any situations in which this has actually occurred in practice, and thus opted for the specification given in the text. Nevertheless, this extension is solved in the appendix, and the results are summarized in the end of this section.

is never surprised by a predecessor's behavior, because he would have acted in exactly the same way.

In solving the dynamic game, we focus on stationary Markov strategies, or those that depend only on the current state of the world. A state is given by a pair $\omega = (R, z) \in \Omega$, where $\Omega = \{S, W\} \times [\underline{z}, \overline{z}]$. A stationary Markov strategy for player 1 consists in the functions $f : \Omega \to (0, \infty)$ and $\lambda : \Omega \to [0, 1]$, with values denoted f_ω and λ_ω, respectively. Here f_ω denotes player 1's license fee offer in state ω, while λ_ω gives the probability that player 1 will litigate following a rejection in state ω.[22] A stationary Markov strategy for player 2 is a function $\alpha : \Omega \times (0, \infty) \to [0, 1]$, with values denoted $\alpha_\omega(f)$, which denotes the probability that player 2 will accept the offer f_ω in state ω. Given the way the impressionable type of player 2 is defined, he always plays the strategy $\overline{\alpha}$, which is defined by

$$\overline{\alpha}_\omega(f) = \begin{cases} \mathbf{1}_{\{f_\omega \leq y(z(\omega))\}} & \text{if } R(\omega) = S \text{ or } z(\omega) \leq 0 \\ 0 & \text{if } R(\omega) = W \text{ and } z(\omega) > 0 \end{cases} \tag{1}$$

where $\mathbf{1}_{\{.\}}$ denotes the indicator function, and where $R(\omega)$ and $z(\omega)$ denote the R and z components of ω, respectively. Given that there are no reputation effects when $z \leq 0$, the players will play the stage game SPNE strategies in any such period in equilibrium. Thus we can impose $f_\omega = y(z(\omega))$, $\lambda_\omega = 1$ and $\alpha_\omega = 1$ whenever $z(\omega) \leq 0$.

We solve the game for stationary Markov perfect equilibria (MPEs) using the dynamic programming approach. Let $V_\omega(\alpha|\alpha^+)$ denote player 1's maximized expected present discounted value of entering a period with state ω, given that: (1) the normal type of player 2, if realized in the present period, will play strategy α; (2) the normal type of player 2, when realized in any subsequent period, will play strategy α^+; and (3) the impressionable type of player 2 will play strategy $\overline{\alpha}$ in any period in which his type is realized. Hence $V_\omega(\alpha|\alpha^+)$ is the expected present discounted value of best-responding to these strategies over time. Note that, under this definition, the continuation value of ensuring a reputation R in the next period is given by δV_R, where $V_R \equiv \mathbb{E}_\Phi[V_{(R,z)}(\alpha^+|\alpha^+)]$, and where $\delta \in (0, 1)$ denotes player 1's intertemporal discount factor. With this, $V_\omega(\alpha|\alpha^+)$ can be defined via the following Bellman equation:

$$\begin{aligned} V_\omega(\alpha|\alpha^+) &= \mathbf{1}_{\{z(\omega) \leq 0\}}(y(z(\omega)) + \delta V_{R(\omega)}) \\ &\quad + \mathbf{1}_{\{z(\omega) > 0\}} \max_{f,\lambda} \Big\{ a_\omega(f, \lambda, \alpha)(f_\omega + \delta V_S) \\ &\quad + \lambda_\omega[1 - a_\omega(f, \lambda, \alpha)](-z(\omega) + \delta V_S) + (1 - \lambda_\omega)[1 - a_\omega(f, \lambda, \alpha)]\delta V_W \Big\} \end{aligned} \tag{2}$$

where $a_\omega(f, \lambda, \alpha) \equiv p\overline{\alpha}_\omega(f) + (1 - p)\alpha_\omega(f)$ gives the probability that player 2 will accept in the current period, given the current state and players' strategies. As for the normal type of player 2, his problem is to minimize his expected loss. It is easy to see that his best response is always given by:

$$\alpha_\omega(f, \lambda) = \mathbf{1}_{\{f_\omega \leq \lambda_\omega y(z(\omega))\}} \tag{3}$$

It is easy to use (2) and (3) to define a stationary MPE in the dynamic game. This is given in Definition 1, below.

[22] We need not consider the possibility that f is randomized, because player 2's best response to it will depend only on the realization of f, not on its mixture.

Definition 1: *A stationary Markov perfect equilibrium of the dynamic game is a profile of stationary Markov strategies* $(f^*, \lambda^*, \alpha^*)$ *such that, for every state* $\omega \in \Omega$*: (i) when* $z(\omega) > 0$*,* f^* *and* λ^* *solve the maximization problem in* (2) *conditional on* $\alpha = \alpha^+ = \alpha^*$*, and* α^* *is consistent with* (3) *conditional on* $(f, \lambda) = (f^*, \lambda^*)$*; and (ii) when* $z(\omega) \le 0$*,* $(f^*, \lambda^*, \alpha^*)$ *forms the subgame perfect Nash Equilibrium of the stage game.*

By inspection of (2) and (3), it is easy to characterize the basic form of equilibrium strategies. Player 1's strategy balances the expected cost of predatory litigation against the discounted incremental value of entering the next period with a strong reputation rather than a weak one. Remark 1 provides a detailed overview of the form of equilibrium strategies.

Remark 1: In any stationary Markov perfect equilibrium, strategies must take the following form:

- Player 1 chooses a litigation threshold $\hat{z} > 0$ and litigates for sure when $z \le \hat{z}$ and otherwise gives up. More specifically, player 1 sets $\hat{z} = \delta(V_S - V_W) > 0$ and $\lambda^*_\omega = \mathbf{1}_{\{z(\omega) \le \hat{z}\}}$ for all ω.[23] Additionally, player 1 offers $f^*_\omega = y(z(\omega))$ for all ω.[24]
- Given f^* and λ^*, the normal type's strategy collapses to $\alpha^*_\omega = \lambda^*_\omega$ for all ω, while the impressionable type's strategy collapses to $\overline{\alpha}_\omega = \mathbf{1}_{\{R(\omega)=S \text{ or } z(\omega) \le 0\}}$ for all ω.

The strategies in Remark 1 generally yield nontrivial dynamics, with player 1 intermittently forfeiting his reputation (when z is sufficiently larger than zero) and rebuilding it (when z is sufficiently low but still positive) over time. When player 1's reputation is weak, he rebuilds it in periods with $z \in (0, \hat{z}]$ by either engaging in predatory patent litigation against an impressionable type (earning a payoff of $-z_t < 0$), or by having his offer accepted by a normal type (earning payoff $y(z)$). To fully characterize the dynamics, let $\theta_t \in \{\theta_0, \theta_I\}$ denote the realization of player 2's type in period t, where θ_0 and θ_I denote the normal and impressionable types, respectively. Also let $u_{1,t}$ denote the payoff earned by player 1 in period t. Table 1 below describes the equilibrium dynamics by giving the outcome vector $(u_{2,t}, R_{t+1})$ as a function of R_t, z_t and θ_t.

Table 1: Equilibrium Dynamics

θ_t	$R_t = S$		$R_t = W$		
	$z_t \le \hat{z}$	$z_t > \hat{z}$	$z_t \le 0$	$0 < z_t \le \hat{z}$	$z_t > \hat{z}$
$\theta_t = \theta_0$	$(y(z_t), S)$	$(0, W)$	$(y(z_t), W)$	$(y(z_t), S)$	$(0, W)$
$\theta_t = \theta_I$	$(y(z_t), S)$	$(y(z_t), S)$	$(y(z_t), W)$	$(-z_t, S)$	$(0, W)$

The strategies in Remark 1 can be defined for arbitrary thresholds $\hat{z} > 0$, not just those in the support of Φ.[25] Given any threshold $\hat{z} > 0$, imposing these strategies on the players yields a dynamic program describing the dynamics of the predatory litigation strategy corresponding to $\hat{z}$. Letting $V^{\hat{z}}_\omega$ denote the value function describing the present discounted value of entering a period

[23] It will be easy to verify that this definition of $\hat{z}$ is indeed positive, reflecting the fact that it is strictly more valuable to have a strong reputation than a weak one.

[24] It is easy to see that this offer is strictly best whenever player 1 actually cares about what offer he makes.

[25] Thresholds $\hat{z} > \overline{z}$ ($\hat{z} < \underline{z}$) correspond to strategies in which player 1 will always (never) litigate following a rejection, given the possible realizations of z.

with state ω, given threshold $\hat{z}$. This program is characterized by the following system of Bellman equations:

$$
\begin{aligned}
V^{\hat{z}}_{(S,z)} &= \mathbf{1}_{\{z\leq\hat{z}\}}[y(z)+\delta V^{\hat{z}}_S] + \mathbf{1}_{\{z>\hat{z}\}}[p(y(z)+\delta V^{\hat{z}}_S) + (1-p)\delta V^{\hat{z}}_W] \\
V^{\hat{z}}_{(W,z)} &= \mathbf{1}_{\{z\leq 0\}}[y(z)+\delta V^{\hat{z}}_W] + \mathbf{1}_{\{0<z\leq\hat{z}\}}[-pz+(1-p)y(z)+\delta V^{\hat{z}}_S] + \mathbf{1}_{\{z>\hat{z}\}}\delta V^{\hat{z}}_W
\end{aligned}
\tag{4}
$$

where $V^{\hat{z}}_R = \mathbb{E}_\Phi[V^{\hat{z}}_{(R,z)}]$ for each $R = S, W$. This program describes the dynamics of an equilibrium if and only if the imposed strategies form a MPE. As indicated by Remark 1, this is so when $\hat{z} = \delta(V^{\hat{z}}_S - V^{\hat{z}}_W)$. To determine when this condition is satisfied it is necessary to define the expectations $V^{\hat{z}}_S$ and $V^{\hat{z}}_W$ expressly. Taking expectations over (4) and grouping terms yields the following Bellman equations:

$$
\begin{aligned}
V^{\hat{z}}_S &= Y(0,\hat{z}) + pY(\hat{z},\overline{z}) + \delta V^{\hat{z}}_S - \delta(1-p)(1-\Phi(\hat{z}))[V^{\hat{z}}_S - V^{\hat{z}}_W] \\
V^{\hat{z}}_W &= Y(\underline{z},0) + (1-p)Y(0,\hat{z}) - pZ(0,\hat{z}) + \delta V^{\hat{z}}_W + \delta(\Phi(\hat{z})-\Phi(0))[V^{\hat{z}}_S - V^{\hat{z}}_W]
\end{aligned}
\tag{5}
$$

$$
\text{where} \quad Y(a,b) \equiv \int_a^b y(z)\phi(z)dz, \qquad Z(a,b) \equiv \int_a^b z\phi(z)dz
$$

Thus $pZ(0,\hat{z})$ gives player 1's unconditional expected cost of predatory litigation in a period with $R = W$. Using (5) it is easy to define the function $\pi(\hat{z}) \equiv \delta(V^{\hat{z}}_S - V^{\hat{z}}_W)$ explicitly. We refer to $\pi(\hat{z})$ as player 1's *reputation premium*. Subtracting $V^{\hat{z}}_W$ from $V^{\hat{z}}_S$ and rearranging yields

$$
\pi(\hat{z}) = \delta p\left(\frac{Y(0,\overline{z}) + Z(0,\hat{z})}{1-\delta[p(1-\Phi(\hat{z}))+\Phi(0)]}\right) \tag{6}
$$

A stationary MPE is characterized by a fixed point $\hat{z}^* = \pi(\hat{z}^*)$. Importantly, an equilibrium may not involve an interior point $\hat{z}^* \in (\underline{z}, \overline{z})$. For example, it may be that player 1 is willing to pay more than any possible realization of z in order to maintain a strong reputation, implying $\hat{z}^* \geq \overline{z}$, in which case player 1 would never give up in equilibrium. As such, it will be necessary to extend the domain of π to include all of $\mathbb{R}$. Clearly π is constant when evaluated outside the interior set $(\underline{z}, \overline{z})$, with $\pi(\hat{z}) = \pi(\underline{z})$ for all $\hat{z} \leq \underline{z}$ and $\pi(\hat{z}) = \pi(\overline{z})$ for all $\hat{z} \geq \overline{z}$. Note also that π is continuous on $\mathbb{R}$, and continuously differentiable everywhere but $\underline{z}$ and $\overline{z}$, which are kink points. Finally, $\pi(\hat{z}) > 0$ for all $\hat{z}$, implying that it is always strictly more valuable to have a strong reputation than a weak one. This reflects the fact that a non-predatory strategy ($\hat{z} = 0$) is never subgame perfect in the dynamic game.[26]

An equilibrium involves predatory litigation if there are possible realizations $z > 0$ at which player 1 strictly prefers to litigate. This is so whenever the equilibrium threshold satisfies $\hat{z}^* > \max\{\underline{z}, 0\}$. In fact, all that is required for this to obtain is that $\underline{z}$ is not too much larger than zero. This is embodied in assumption (A1) below.

[26] To see this, suppose that player 1 is engaged in a non-predatory strategy ($\hat{z} = 0$) and has a weak reputation. Then suppose he observes a very low (but positive) realization of z, and consider a one shot deviation to a predatory strategy under which he would litigate in present period. If he deviates for this period only, his payoff in the next period increases in expectation by $pY(0,\overline{z})$ (his continuation value would also be higher in the next period). Thus, if $0 < z < \delta pY(0,\overline{z})$ the deviation is strictly profitable.

$$\underline{z} < \frac{\delta p}{1 - \delta p} \mathbb{E}[y(z)] \tag{A1}$$

Clearly (A1) holds whenever $\underline{z} \leq 0$, in which case the positivity of π implies that any equilibrium must involve predatory litigation. When $\underline{z} > 0$, (A1) is needed to ensure equilibrium predation. Noting that the righthand side of (A1) is equal to $\pi(\underline{z})$ when $\underline{z} \geq 0$, (A1) simply ensures that player 1 has a strict preference to engage in predatory patent litigation at the lowest possible realizations of $z > 0$. Finally, in establishing this section's primary equilibrium result, the following condition will prove invaluable in ascertaining the shape of π.

$$\text{sign}\left\{ \frac{\partial \pi(\hat{z})}{\partial \hat{z}} \right\} = \text{sign}\left\{ \hat{z} - \pi(\hat{z}) \right\} \qquad \forall \hat{z} \in (\underline{z}, \overline{z}) \tag{$\star$}$$

Among other things, condition ($\star$) establishes that the restriction $\pi|_{(\underline{z},\overline{z})}$ can change from decreasing to increasing (or vice versa) only at a fixed point. Thus, if $\pi|_{(\underline{z},\overline{z})}$ has a fixed point $\hat{z}^*$, then it has a U-shape with a minimum point at $\hat{z}^*$. With this, Proposition 1 establishes this paper's primary equilibrium result. Figure 2 below illustrates the equilibrium result for the interior case $\hat{z}^* \in (\underline{z}, \overline{z})$ with $\underline{z} < 0$.

Proposition 1: *There exists a unique stationary Markov perfect equilibrium of the dynamic game, and it involves predatory litigation if and only if* (A1) *holds.*

Proof: Appendix.

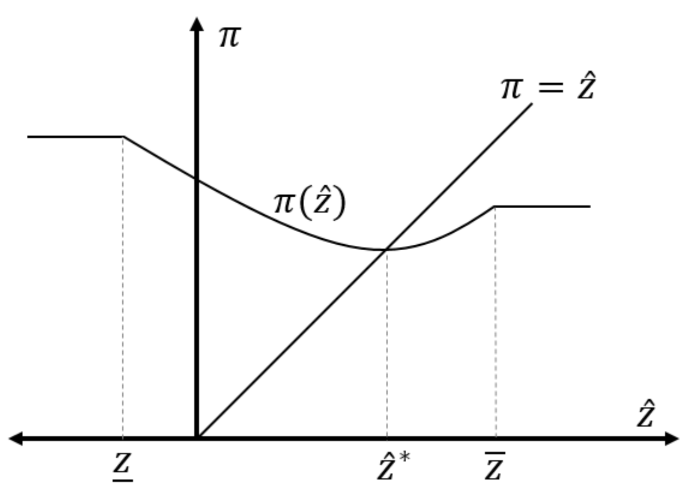

Figure 2: Interior Equilibrium

Before concluding this section, it is worth addressing a possible extension, which was considered in footnote 21. It involves adjusting the model to allow player 1 to garner a positive reputation effect when $z \leq 0$ by offering an unacceptably high license fee in order to force unnecessary litigation. This has the same aggressive flavor as predatory litigation, since player 1 could have garnered a higher payoff by simply offering $y(z)$. This extension is solved in the appendix, and the results are intuitive: as before, player 1 chooses a litigation threshold $\hat{z} > 0$ (defined in the same way) and always litigates following a rejection when $z \leq \hat{z}$. But now player 1 also chooses a nonpositive threshold $z_0 \leq 0$ such that, when his his reputation is weak, he offers an unacceptably high fee and litigates when $z \leq z_0$. This makes a weak reputation comparatively less harmful for player 1, as it is now easier to rebuild a strong reputation. This works to create a downward shift in the reputation premium, thereby weakly reducing the extent of equilibrium predatory litigation (i.e. $\hat{z}^*$ falls). However, this does not necessarily suggest that welfare increases, because the unnecessary litigation player 1 forces under this strategy is socially costly in its own right.

2.1 Fee Shifting Rules

One measure often proposed to combat frivolous patent litigation is the expanded use of fee shifting, which is rule under which a losing party may be made to pay the prevailing party's attorney's fees after trial. American Courts generally do not shift fees. And, while §85 of the Patent Act expressly permits fee shifting in "exceptional cases," this provision was historically interpreted narrowly, and as a consequence it was very rarely invoked.[27] However, a recent Supreme Court decision relaxed this rigid interpretation of §85, holding that "an 'exceptional case' is simply one that stands out from the others with respect to the substantive strength of a partys litigating position."[28] Additionally, some recently proposed bills, such as the Innovation Act, would create a new statutory basis for expanded fee shifting.[29] These measures would provide broader discretion for the courts to shift attorney's fees when a litigant's position seems particularly weak.

A fee shifting rule is characterized by a pair (z^+, σ). Here $z^+ \geq 0$ is a threshold such that, if player 1 loses in court, player 2's fees are shifted if $z > z^+$, but not shifted if $z \leq z^+$. Hence z^+ specifies what infringement claims are of sufficiently low quality to merit fee shifting. The assumption that z^+ is nonnegative embodies the fact that the aforementioned fee shifting standards do not call for universal fee shifting, but rather take effect only when a litigant's claim seems to be relatively bad.[30] We assume that $z^+ < \overline{z}$, because the extent of equilibrium predatory litigation will not change if $z^+ \geq \overline{z}$. The function $\sigma(z)$ gives the expected fee shifting payment player 1 will have to make to player 2 in a period with $z > z^+$. Importantly, a fee shifting payment will generally not fully compensate the defendant for the injury he suffers as a result of litigation. Diverting resources to finance litigation will often impose a number of ancillary harms and opportunity costs on the defendant's business operations. To capture this, we decompose player 2's litigation injury as follows:

[27] The Patent Act's fee shifting provision is codified in 35 U.S.C. §285. For an illustration of the previously rigid approach to fee shifting, see See *Brooks Furniture Mfg., Inc. v. Dutailier Intl, Inc.*, 393 F. 3d 1378 (Fed. Cir. 2005).

[28] See *Octane Fitness, LLC v. Icon Health & Fitness, Inc.*, No. 12-1184 (2014)

[29] The proposed Innovation Act is codified in H.R. 3309.

[30] Alternatively, if there is fee shifting even when $z < 0$, then our results are even stronger: fee shifting does even less to mitigate predatory litigation. This is because a fee shifting rule that applies even when $z < 0$ would cause litigation to have negative expected value even for some negative z values. Thus litigation would less often be subgame perfect in the stage game. Given that reputation only matters when litigation is not subgame perfect in the stage game, this causes player 1 to place more value on reputation, thereby placing upward pressure on the reputation premium. This is a separate effect of fee shifting that works against the effects addressed in the text, and thus a fee shifting rule of this type would have a smaller effect on predatory litigation than that identified under the assumption $z^+ \geq 0$.

$$y(z) = y_L(z) + y_A(z, v) + \rho(z)m \tag{7}$$

Here $y_L(z)$ denotes player 2's direct cost of litigation (attorney's fees), which is nonincreasing with $y_L(z) > 0$ for all z. $\rho(z)$ gives the probability that player 1 wins the lawsuit, and is nonincreasing with $\rho(z) < 1$ for all z. $m > 0$ denotes the expected monetary damages player 2 will have to pay player 1, conditional on player 1 winning in court. Finally, $y_A(z, v)$ denotes the *ancillary harm* suffered by player 2 as a result of having to shift resources to litigation. This includes any harm imposed on player 2's business operations as a result of its engagement in litigation. y_A is nonincreasing in z and strictly increasing in v, with $y_A(z, v) > 0$ for all z and v. Here $v \in \mathbb{R}$ is a parameter describing player 2's *vulnerability*. A more vulnerable defendant is one who suffers a larger ancillary harm in any given lawsuit. For a large firm with ample resources, v may be low, and thus $y_A(z, v)$ may be relatively small, because the firm can manage to litigate the suit without having to make any substantive sacrifices in its ongoing business operations. But for a small, cash-strapped defendant, litigation may have a crippling effect, and thus y_A may be very large. For example, litigation may force the firm into bankruptcy. Alternatively, it may force the firm to pass on valuable opportunities for company growth, to lay off valuable employees, or to invest less in research or product improvements, causing the firm to fall behind competitors. Using (7), we can define $\sigma(z)$ as

$$\sigma(z) = [1 - \rho(z)]y_L(z)$$

which satisfies $0 < \sigma(z) < y(z)$ for all z. Given that ρ and y_L are nonincreasing functions, it is ambiguous whether σ is increasing or decreasing in z, or whether it is even monotone. We thus elect to simplify the problem by assuming it is constant with $\sigma(z) = \sigma$ for all z, implying that $\sigma \in (0, y(\overline{z}))$.

It is easy to look at player 1's decision problem in (2) and see how things change under a fee shifting rule. Let $\pi^*_\sigma = \delta(V_{S,\sigma} - V_{W,\sigma}) > 0$ denote the reputation premium in an equilibrium under fee shifting. Analogous to V_S and V_W in (2), $V_{R,\sigma}$ denotes the equilibrium expected present discounted value of entering a period with reputation R. Given any realization z, player 1's expected litigation injury is $\psi_\sigma(z) = z + \mathbf{1}_{\{z>z^+\}}\sigma$, which is strictly increasing with a discontinuous upward jump at z^+. Player 1's strategy is to litigate following a rejection whenever $\psi_\sigma(z) \leq \pi^*_\sigma$. This involves setting a litigation threshold $\hat{z}$ such that $\psi_\sigma(\hat{z} - \varepsilon) < \pi^*_\sigma < \psi_\sigma(\hat{z} + \varepsilon)$ for all $\varepsilon > 0$. If there is a solution to $\psi_\sigma(\hat{z}) = \pi^*_\sigma$, then this solution is the equilibrium threshold. However, if ψ_σ jumps over π^*_ω at the discontinuous point z^+, meaning that $z^+ < \pi^*_\sigma \leq z^+ + \sigma$, then the equilibrium threshold is $\hat{z} = z^+$. Thus, an equilibrium is characterized by an intersection $\pi^*_\omega \in \Psi_\sigma(\hat{z})$, where Ψ_σ is the correspondence defined by

$$\Psi_\sigma(\hat{z}) = \begin{cases} \hat{z} & \text{if } \hat{z} < z^+ \\ [z^+,\, z^+ + \sigma] & \text{if } \hat{z} = z^+ \\ \hat{z} + \sigma & \text{if } \hat{z} > z^+ \end{cases}$$

Thus, as before, the litigation component of player 1's strategy takes the form $\lambda^*_\omega = \mathbf{1}_{\{z(\omega)\leq\hat{z}\}}$. However, equilibrium license offers and acceptance decisions change in some periods, reflecting the fact that fee shifting changes players' litigation injuries in periods with $z > z^+$. In these periods, player 1's expected litigation injury increases to $z + \sigma$, while player 2's falls to $y(z) - \sigma$. We assume that the impressionable type understands the impact of fee shifting on his expected litigation injury, and thus neither type will accept any offers exceeding $y(z) - \sigma$ in these periods.

As a result, player 1's equilibrium offer becomes $f^*_\omega = y(z(\omega)) - \mathbf{1}_{\{z(\omega)>z^+\}}\sigma$ for every ω. With this, acceptance decisions along the equilibrium path are exactly as before: in any state ω, the normal type accepts if and only if $z(\omega) \leq \hat{z}$, and the impressionable type accepts if and only if either $R(\omega) = S$ or $z(\omega) \leq 0$.

As before, we can impose strategies taking the equilibrium form for arbitrary thresholds $\hat{z} > 0$ in order to obtain a dynamic program. Analogous to the the value functions in (5), we let $V^{\hat{z}}_{R,\sigma}$ denote the expected present discounted value of entering a period with reputation R under a fee shifting rule, given the strategies characterized by threshold $\hat{z}$. $V^{\hat{z}}_{S,\sigma}$ and $V^{\hat{z}}_{W,\sigma}$ are given by

$$\begin{aligned} V^{\hat{z}}_{S,\sigma} &= Y(0,\hat{z}) + pY(\hat{z},\overline{z}) - p\Sigma(\hat{z}) - \mathbf{1}_{\{\hat{z}>z^+\}}[\Phi(\hat{z}) - \Phi(z^+)]\sigma \\ &\quad + \delta V^{\hat{z}}_{S} - \delta(1-p)(1-\Phi(\hat{z}))[V^{\hat{z}}_{S,\sigma} - V^{\hat{z}}_{W,\sigma}] \\ \\ V^{\hat{z}}_{W,\sigma} &= Y(\underline{z},0) + (1-p)Y(0,\hat{z}) - \mathbf{1}_{\{\hat{z}>z^+\}}[\Phi(\hat{z}) - \Phi(z^+)]\sigma - pZ(0,\hat{z}) \\ &\quad + \delta V^{\hat{z}}_{W} + \delta(\Phi(\hat{z}) - \Phi(0))[V^{\hat{z}}_{S,\sigma} - V^{\hat{z}}_{W,\sigma}] \end{aligned} \tag{8}$$

$$\text{where} \quad \Sigma(\hat{z}) \equiv \big[1 - \max\{\Phi(\hat{z}), \Phi(z^+)\}\big]\sigma$$

This program characterizes an equilibrium if the imposed strategies form a stationary MPE. This occurs at an intersection $\pi_\sigma(\hat{z}^*_\sigma) \in \Psi_\sigma(\hat{z}^*_\sigma)$, where $\pi_\sigma(\hat{z}) = \delta(V^{\hat{z}}_{S,\sigma} - V^{\hat{z}}_{W,\sigma})$ is the reputation premium under fee shifting, given the strategies characterized by $\hat{z}$. Using (8), π_σ is given by

$$\pi_\sigma(\hat{z}) = \delta p\left(\frac{Y(0,\overline{z}) + Z(0,\hat{z}) - \Sigma(\hat{z})}{1 - \delta[p(1-\Phi(\hat{z})) + \Phi(0)]}\right) \tag{9}$$

π_σ is positive-valued, because $\sigma \in (0, y(\overline{z}))$ and $z^+ \geq 0$ imply that $Y(0,\overline{z}) > \Sigma(\hat{z})$. Note that π_σ inherits the kink point in Σ, and is thus non-differentiable at z^+. Additionally, it is easy to verify that π_σ satisfies the following condition, which is analogous to ($\star$):

$$\text{sign}\left\{\frac{\partial \pi_\sigma(\hat{z})}{\partial \hat{z}}\right\} = \begin{cases} \text{sign}\{\hat{z} - \pi_\sigma(\hat{z})\} & \text{if } \hat{z} \in (\underline{z}, z^+) \\ \text{sign}\{\hat{z} + \sigma - \pi_\sigma(\hat{z})\} & \text{if } \hat{z} \in (z^+, \overline{z}) \end{cases} \tag{$\star\star$}$$

As before, all that is needed for an equilibrium to involve predatory litigation is that $\pi_\sigma(\underline{z}) > \underline{z}$, which will imply that there are possible realizations $z > 0$ at which player 1 strictly prefers to engage in predatory litigation. This condition is given in assumption (A2).

$$\underline{z} < \frac{\delta p}{1 - \delta p}[\mathbb{E}[y(z)] - \Sigma(\underline{z})] \tag{A2}$$

Assumption (A2) is slightly more restrictive than (A1), but it still allows for positive values of $\underline{z}$ to generate predatory litigation in equilibrium. Proposition 2 establishes that there exists a unique stationary MPE under fee shifting, and that fee shifting weakly reduces the extent of equilibrium predatory litigation, meaning that $\min\{\hat{z}^*_\sigma, \overline{z}\} \leq \min\{\hat{z}^*, \overline{z}\}$, where $\hat{z}^*$ is the equilibrium litigation threshold without fee shifting.

Proposition 2: *Under fee shifting, there exists a unique stationary Markov perfect equilibrium of the dynamic game, and it involves predatory litigation if and only if* (A2) *holds. Moreover, fee* *shifting weakly reduces the extent of equilibrium predatory litigation.*

Proof: Appendix

Thus, a fee shifting rule will generally be insufficient to eliminate predatory patent litigation. It may reduce the extent of predatory litigation, but it will generally not deter it entirely. However, if we depart from this model's rigid assumption that player 1 cannot discriminate among different potential litigation targets, then even this limited improvement is not certain to occur. In particular, if the PAE has some flexibility in choosing the targets of his patent assertion, then fee shifting may simply convince the PAE to choose more vulnerable defendants, i.e. those with a higher value of v. All else equal, a plaintiff can extract a larger amount in expectation from a more vulnerable defendant. A fee shifting rule only strengthens the incentive to target these defendants, because a fee shifting payment does not include any compensation for the ancillary harms imposed on the defendant's business operations. And, importantly, these ancillary harms are just as relevant as attorney's fees in determining what amount a potential defendant is willing to spend to avoid litigation. Thus a strategy of targeting more vulnerable defendants allows a PAE to impose a larger litigation injury without increasing his expected fee shifting payment. This approach will be particularly successful when employed against small innovative firms, such as technology startups, because these firms sustainability will frequently hinge on their ability to rapidly innovate in order to stay ahead of the competition. And, given that many such firms offer novel products not available from other sellers, the external impact of predatory litigation on consumers may particularly acute when these firms are targeted. For these reasons, it seems unlikely that a fee shifting rule will provide an adequate solution to the problem of predator patent litigation.

3 Innovation and Patenting Incentives

The foregoing analysis demonstrates that predatory patent litigation is a profitable means of extracting royalties for low quality patents that would otherwise be difficult or impossible to license. It also shows that a fee shifting rule is unlikely to solve the problem. It will generally not be a sufficiently strong deterrent, and predatory PAEs can further dampen its punitive impact by targeting smaller, more vulnerable defendants for whom litigation will prove crippling even if attorney's fees are ultimately recouped after trial. The next step in the analysis centers on the impact of predatory patent litigation on the innovation and patenting decisions of prospective inventors.

As Hagiu and Yoffie (2012) note, the principal argument supporting PAEs' impact on innovation is that they create an outlet for small-scale inventors to monetize their ideas. The argument here is that PAEs bolster the incentive to innovate among small inventors who would otherwise be unable to net a sufficient return on their efforts because they lack the resources to enforce their own patents. However, even if this argument is sound, there is reason to doubt that it applies to predatory PAE conduct. By definition, predatory patent litigation surrounds low quality patents – arguably infringed but very likely invalid. To the extent that lower quality patents correspond to less valuable technological advancements, predatory patent litigation could only encourage development of ideas that provide little or no incremental social value relative to the existing stock of technologies. Non-predatory PAE activity – buying and asserting comparatively strong patents

– would be sufficient to induce development of those ideas actually making a material contribution.

Additionally, by its nature, predatory patent litigation involves no promulgation of new ideas. The model developed in this paper involves a particular approach to PAE patent holdership. The PAE accuses firms of having already infringed its patents by the time it initiates licensing negotiations. The asserted patents are hypothesized to be of low quality, which typically means that they are excessively broad or that they cover technologies that are relatively obvious or non-novel. Such patents are likely to be widely unintentionally infringed, meaning that the infringement is a product of independent invention of some technology arguably covered by the patent. In this way, the PAE's actions embody what we call a *wait and sue* approach to patent licensing. Under this strategy, the PAE waits until it identifies firms that have unintentionally infringed, at which point it steps in and threatens litigation. This is in stark contrast to the alternative approach of *upfront licensing* in which the PAE works to promulgate the patented technology and reach a wide base of potential users in the hope that they will choose to adopt and license it.

A patent holder will generally be able garner a higher licensing fee from a firm that is already infringing than from a potential user who has not yet adopted the patented technology. The former has already sunk the fixed costs needed to implement the technology, making its continued use more profitable at the margin, and the patent holder can capture this increase. However, this does not imply that the wait and see approach is always best, because this strategy's overall profitability depends on the number of firms that are likely to unintentionally infringe. Thus, if a new technology is legitimately novel and nonobvious, then the PAE's optimal approach is almost certainly upfront licensing. By definition, such a technology is something that other firms are not yet using, and which most potential users are unlikely to come up with on their own. Consequently most potential users are unlikely to even think of adopting the technology unless they are expressly made aware that someone has invented it. As a result, there will be relatively little unintentional infringement, leaving few firms to target under a wait and see approach. By contrast, if a patent is non-novel or obvious, then a wait and sue approach will frequently be most profitable, because many firms will unintentionally infringe the technology, providing the PAE with a large number of targets.

The patent system is intended to induce development of technologies that would not come to market unless their inventors were afforded some protection against competing users of their ideas. Thus a technology should not be patent-protected if it is likely to come to market even if the applicant were denied a patent. This is a clear indication that the technologies disputed in predatory patent litigation ought not to have been patented in the first place. Indeed, the predatory PAE relies on patents covering those technologies most likely to be independently infringed, implying that the technologies would have been widely implemented even if they had been denied patent protection. Hence there is no reason to believe that fewer valuable technologies would come to market if predatory patent litigation were eradicated.

On the other hand, one significant effect of predatory patent litigation is that it induces firms and inventors to do a lot more questionable patenting. Predatory patent litigation creates a market for low quality patents, encouraging applicants to seek excessively broad patens, or patents covering relatively non-novel or obvious technologies. This exacerbates the "patent thicket" problem, which Shapiro (2001) defines as "a dense web of overlapping intellectual property rights that a company must hack its way through in order to actually commercialize new technology." This makes it risky for an innovative firm to bring new technologies to market, as they might arguably be covered by some dubious or excessively broad patents. It also forces inventors to pay for technologies that they came up with autonomously, and which did not merit patent protection in the first place. All of this works to make the competitive environment more risky and expensive for firms and innovators, undermining the very interest the patent system seeks to protect. As a consequence, it is difficult to conceive of any coherent argument for the proposition that predatory patent litigation might produce some pro-competitive effects.

4 Litigation Cost-Sharing Agreements

Although fee shifting is unlikely to adequately deter predatory patent litigation, potential defendants can likely accomplish this through collective action. In particular, potential defendants can form a *litigation cost-sharing agreement* (LCSA), which we define as a contractual arrangement such that, when a member is sued for patent infringement, all members jointly pay its litigation costs as they arise, provided that (1) the infringement claim exhibits some contractually specified characteristics aimed at identifying predatory claims, or it is deemed to be sufficiently unlikely to succeed on the merits by an impartial arbiter; and (2) the defendant in the suit agrees not to settle. If these conditions are not satisfied, then the defendant must finance his own litigation as usual. This functions much like an insurance plan. Each member internalizes only a small share of its defense costs, and is therefore much less daunted by the prospect of litigation. Importantly, an LCSA will greatly diminish the ancillary litigation harms imposed on the defendant's business operations which, as noted in section 2.1, are central to the profitability of predatory patent litigation. Further, by prohibiting settlement, a LCSA maximizes the expected losses a predatory PAE must incur at trial. To the author's knowledge, this is the first proposal of such an arrangement, although other forms of joint defense agreements have been implemented by some defendants.[31]

One obvious concern is that LCSAs might arouse antitrust scrutiny. Plaintiffs might allege that an LCSA constitutes a conspiracy to restrain trade. However, as defined, an LCSA includes safeguards intended to limit its scope. It is not simply an agreement not to license patents. Rather, it is intended to foster risk sharing by protecting members from having to incur substantial litigation costs in order to demonstrate that an asserted patent is in fact invalid. Hovenkamp, Janis and Lemley (2010) note that agreements formed in anticipation of litigation are generally lawful if formed in good faith – i.e. if the parties have an objectively reasonable anticipation of defeating the claim. Given an LCSA's narrowed focus on predatory litigation, such agreements are very likely to allay antitrust concerns on these grounds, because an LCSA's provisions will not kick in if the plaintiff's infringement claim is meritorious. As a consequence, if properly drafted, a LCSA will not deter patent holders from filing good faith infringement claims against member firms. This also ensures that the LCSA will not create a moral hazard problem under which the agreement's protections induce member firms to engage in widespread willful infringement of strong patents.

[31]This typically involves a group of firms who are all accused of infringing a common patent, and who elect to split cost or hire a single counsel. Importantly, these do not involve a requirement that the infringement claim possesses little merit.

Two legal doctrines will enhance the efficacy of LCSAs. The first is the doctrine of issue preclusion, also known as collateral estoppel, which works to prevent the same issue from being litigated twice. In our context, this effectively allows a finding of patent invalidity to carry over to subsequent litigation; future defendants need not reestablish the patent's invalidity.[32] As a result, a finding of invalidity will tend to create a positive externality for other potential defendants. However, one qualification of this doctrine is that the original suit must have been litigated to judgment. This reinforces the notion that potential defendants might benefit from a concerted effort to litigate dubious infringement claims to judgment. Second, following a recent Supreme Court decision,[33] licensees are permitted to challenge the validity of their licensed patents. This gives existing licensees an inducement to contribute to the defense of firms accused of infringing the licensed patents, as they can use a finding of invalidity to terminate their own licensing obligations.

5 Conclusion

Predatory patent litigation allows a PAE to monetize low quality patents that would otherwise be difficult or impossible to license. For a PAE with low quality patents, this provides a litigious reputation that convinces other firms that the PAE's litigation threats are credible, despite their apparent futility. Like predatory pricing, this strategy involves a short run loss that is recouped over time through supra-competitive pricing. Predatory patent litigation imposes substantial costs without doing anything to promote innovation. To the contrary, it actually makes the competitive environment more hazardous for firms and inventors by encouraging applications for excessively broad patents, or those covering technologies so obvious or non-novel that they are likely to be widely independently invented. Importantly, not all PAEs engage in predatory patent litigation, and hence this these results do not support a categorical condemnation of PAE activity at large. This study nevertheless advances the PAE debate by identifying a particular species of PAE whose conduct is unambiguously harmful, and establishes some useful bases for distinguishing among different PAE licensing strategies.

Fee shifting rules will provide some relief, but are unlikely to provide a satisfactory resolution to the predatory litigation problem. The problem is that many defendant-firms suffer litigation injuries substantially exceeding the direct cost of paying attorneys, which is particularly true for small, cash-strapped firms like technology startups. Fee shifting will not compensate these firms for the ancillary harms imposed on their business operations, and hence predatory litigation can still intimidate these firms into settlement. As such, fee shifting will tend to increase a predatory plaintiff's incentive to target smaller, more vulnerable firms for whom these ancillary harms are particularly acute. This paper proposes that potential defendants could better protect themselves by entering into a litigation cost-sharing agreement. Under this arrangement, a member's litigation costs are split among the group as they arise, provided that (1) the infringement suit satisfies some predetermined conditions aimed at identifying predatory lawsuits, or is deemed sufficiently unlikely to win on the merits by an impartial arbiter; and (2) the defendant-member agrees not to settle. If properly limited in scope, such an arrangement will allay antitrust concerns, and will not undermine meritorious infringement claims aimed at member firms.

[32] *Blonder-Tongue Labs, Inc. V. Univ. of Illinois*, 402 U.S. 313 (1971).

[33] *Medimmune, Inc. V. Genentech, Inc.*, 549 U.S. 118 (2007).

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Appendix: Proofs and Extensions

Proposition 1: *There exists a unique stationary Markov perfect equilibrium of the dynamic game, and it involves predatory litigation if and only if* (A1) *holds.*

Proof: We begin by detailing a few properties of π. It is clearly continuous, and it is constant when evaluated outside the interior set $(\underline{z}, \overline{z})$. Additionally, $(\star)$ implies that the restriction $\pi|_{(\underline{z},\overline{z})}$ is strictly decreasing (increasing) when strictly above (below) the 45-degree line. Thus, if $\pi|_{(\underline{z},\overline{z})}$ has a fixed point $\hat{z}^*$, then it has a U-shape with a minimum point at $\hat{z}^*$. Additionally, it is easy to establish that π cannot cross (or simply touch) the 45-degree line at any $\hat{z} \in \mathbb{R}$. To verify this, first note the obvious fact that a differentiable function can cross (or simply touch) the 45-degree line from below at a given point only if the slope at that point is weakly greater than 1. Thus π cannot cross the 45-degree line from below on $(-\infty, \underline{z})$ or $(\overline{z}, \infty)$, since it is differentiable with slope 0 over these ranges. This also rules out the possibility that π crosses from below at any point in the interior set $(\underline{z}, \overline{z})$, because $(\star)$ establishes that π has slope zero at any such intersections. Additionally, π

cannot hit the 45-degree line from below at $\underline{z}$, as it is continuous and initially lies above the 45-line (because $\pi(\hat{z}) > \hat{z}$ for all negative $\hat{z}$). The final possibility is that π hits the 45-degree line from below at $\overline{z}$. To rule this out, it is easy to verify that $\partial\pi(\hat{z})/\partial\hat{z} < H\cdot|\hat{z}-\pi(\hat{z})|$ for all $\hat{z} \in (\underline{z}, \hat{z})$, where

$$H \equiv \frac{\delta p}{1-\delta} \sup_z \phi(z) > 0$$

The assumptions on ϕ ensure that H is well defined and finite. Thus there exists $\Delta > 0$ such that $\partial\pi(\hat{z})/\partial\hat{z} < 1$ whenever $\hat{z} - \pi(\hat{z}) < \Delta$. If in fact π hits the 45-degree line from below at $\overline{z}$, then continuity implies that there exists $\varepsilon > 0$ such that $\hat{z} - \pi(\hat{z}) \in (0, \Delta)$ for all $\hat{z} \in (\overline{z} - \varepsilon, \overline{z})$. But then the difference $\hat{z} - \pi(\hat{z})$ is positive and strictly increasing over $(\overline{z} - \varepsilon_1, \overline{z})$, and thus π is actually diverging from the 45-degree line from below (although still increasing) over this range. Thus, as $\hat{z}$ approaches $\overline{z}$, π must converge to a point strictly below the 45-degree line, which contradicts $\pi(\overline{z}) = \overline{z}$. Thus π cannot cross or touch the 45-degree line from below at any point in $\mathbb{R}$.

Establishing existence of a stationary MPE is trivial, because $\pi(\hat{z}) \in (0, M]$ for all $\hat{z}$, where $M \equiv \max_{\hat{z}\in[\underline{z},\hat{z}]} \pi(\hat{z}) \in (0, \infty)$. Thus π must hit the 45-degree line at some point in $(0, M]$. As such, there must exist a stationary MPE characterized by some $\hat{z}^* > 0$, regardless of whether (A1) holds.

To establish uniqueness, there are two cases to consider. For case 1, suppose that (A1) holds. This implies that $\pi(\underline{z}) > \underline{z}$, because the righthand side of (A1) is equal to $\pi(\underline{z})$ when $\underline{z} \geq 0$, and the positivity of π ensures that $\pi(\underline{z}) > \underline{z}$ whenever $\underline{z} \leq 0$. Given this, consider the two possibilities $\pi(\overline{z}) < \overline{z}$ and $\pi(\overline{z}) \geq \overline{z}$. If $\pi(\overline{z}) < \overline{z}$, then $\pi(\underline{z}) > \underline{z}$ implies π must cross the 45-degree line from above at some $\hat{z}^* \in (\underline{z}, \overline{z})$. There can be only one such fixed point, because π cannot cross the 45-degree line from below, which also implies that there is no fixed point larger than $\hat{z}^*$. Additionally there are no fixed points lower than $\underline{z}$, because $\pi(\hat{z}) = \pi(\underline{z}) > \underline{z} \geq \hat{z}$ for all $\hat{z} \leq \underline{z}$. Thus $\hat{z}$ is unique, and satisfies $\hat{z} > \max\{\underline{z}, 0\}$. Alternatively, if $\pi(\overline{z}) \geq \overline{z}$, then it must be that $\pi|_{[-\infty,\hat{z})}$ lies strictly above the 45-degree line, given that π cannot cross or touch the 45-degree line from below. Thus there are no fixed points strictly lower than $\overline{z}$. Additionally, there must be a unique fixed point $\hat{z}^* \in [\overline{z}, \infty)$, because π is constant (and initially (weakly) above the 45-degree line) over this range. Thus there is again a unique fixed point with $\hat{z}^* \max\{\underline{z}, 0\}$.

For case 2, suppose that (A1) fails. Then $\underline{z} > 0$ and π lies weakly below the 45-degree line at $\underline{z}$. Given that $\pi(\hat{z}) = \pi(\underline{z}) > 0 \geq \hat{z}$ for all $\hat{z} \leq 0$, this implies that π must intersect the 45-degree line at a point $\hat{z}^* \in (-\infty, \underline{z}]$. This is the only fixed point in that range, because π is constant over that interval. Additionally, there can be no fixed points strictly larger than $\underline{z}$, because π cannot cross the 45-degree line from below.

Thus, there is always a unique stationary MPE. However, note that in case 2, the equilibrium had $\hat{z}^* \not> \max\{\underline{z}, 0\}$, and thus there is no predatory litigation in equilibrium. By contrast, in case 1 we found that the equilibrium necessarily involved predatory litigation. Hence the equilibrium involves predatory litigation if and only if (A1) holds, as desired.

■

Proposition 2: *Under fee shifting, there exists a unique stationary Markov perfect equilibrium of the dynamic game, and it involves predatory litigation if and only if* (A2) *holds. Moreover, fee* *shifting weakly reduces the extent of equilibrium predatory litigation.*

Proof: Note that, similar to π, π_σ is continuous and positive-valued on $\mathbb{R}$, and it is constant when evaluated outside the interior set $(\underline{z}, \overline{z})$. Additionally, given $(\star\star)$, the same argument given in the proof of proposition 1 (henceforth "proof 1") implies that π_σ cannot cross or touch Ψ_σ from below. In particular, the restriction $\pi_\sigma|_{(-\infty,z^+)}$ cannot hit the 45-degree line from below (nor can it attain $\pi_\sigma(z^+) = z^+$ from below) and, by analogy, $\pi_\sigma|_{(z^+,\infty)}$ cannot hit the line $\hat{z} + \sigma$ from below. Again following proof 1, there must always be an equilibrium characterized by some threshold $\hat{z}^*_\sigma > 0$.

Let $\hat{z}^*_\sigma > 0$ satisfy $\pi_\sigma(\hat{z}^*_\sigma) \in \Psi_\sigma(\hat{z}^*_\sigma)$, so that it corresponds to a stationary MPE. Suppose first that (A2) holds. Analogous to (A1) in proof 1, this implies that $\pi_\sigma(\underline{z}) > \underline{z}$. If $\underline{z} < z^+$ and $\hat{z}^*_\sigma \in (\underline{z}, z^+)$ then, following the arguments in proof 1, this is the only intersection point over this range, and there are no intersection points in $(-\infty, \underline{z}]$. Additionally, π_σ cannot hit ψ_σ from below, and thus there are no intersection points larger than $\hat{z}^*_\sigma$, which is therefore unique. The case for $\hat{z}^*_\sigma \in (z^+, \infty)$ similarly follows by analogy from proof 1. This case also establishes the existence and uniqueness of an equilibrium when $\underline{z} \geq z^+$, because in that case $\pi_\sigma(\underline{z}) > \underline{z}$ implies that $\hat{z}^*_\sigma > z^+$. If $\hat{z}^*_\sigma = z^+$, then $\pi_\sigma(\underline{z}) > \underline{z}$ and $(\star\star)$ imply that $\pi_\sigma(\hat{z}) > \hat{z}$ for all $\hat{z} < z^+$. And there can be no intersection point larger than $\hat{z}^*_\sigma$, because $\pi_\sigma|_{(z^+,\infty)}$ will not touch Ψ_σ from below. Note that in all of these cases the equilibrium threshold is unique and satisfies $\hat{z}^*_\sigma > \max\{\underline{z}, 0\}$.

Alternatively, if (A2) fails, then the argument is again exactly analogous to that given in proof 1. π_σ must cross Ψ_σ at some $\hat{z}^*_\sigma \leq \underline{z}$, and it cannot cross Ψ_σ from below, so there are no other intersection points, implying $\hat{z}^*_\sigma$ is the unique fixed point. And this intersection point does not involve predatory litigation, confirming that an equilibrium in the presence of fee shifting involves predatory litigation if and only if (A2) holds. Finally, note that that if $z^+ < \overline{z}$, then $\pi(\hat{z}) > \pi_\sigma(\hat{z})$ for all $\hat{z}$. Consequently, given that Ψ_σ lies weakly above the 45-degree line (strictly for $\hat{z} > z^+$), it follows that $\pi(\hat{z}^*_\sigma) > \hat{z}^*_\sigma$ (weakly if $z^+ < \overline{z}$). Thus π intersects the 45-degree line at a threshold higher than $\hat{z}^*_\sigma$, implying that fee shifting reduces the equilibrium litigation threshold. However, depending on the support of Φ, this does not necessarily mean that less predatory litigation takes place in equilibrium, because it could be that $\hat{z}^*_\sigma \geq \overline{z}$, in which case fee shifting has no effect on the prevalence of predatory litigation.

■