

# A Model of Optimal Government Bailouts

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## Abstract

We analyze incentive-efficient government bailouts within a canonical model of intra-firm moral hazard. Bailouts exacerbate the moral hazard of firms and managers in two ways. First, they make them less averse to failing. Second, the taxes to fund bailouts dampen their incentives. Nevertheless, if third-party externalities from keeping the firm alive are strong, bailouts can improve welfare. Our model suggests that governments should use bailouts sparingly, where social externalities are large and subsidies small; eliminate incumbent owners and managers to improve *a priori* incentives; and finance bailouts through redistributive taxes on productive firms instead of forcing recipients to repay in the future.

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Prior to 2007, most academic economists viewed government bailouts as aberrations of developing countries, artifacts of political patronage, or idiosyncrasies of the banking industry. The consensus view of governmental interventions generally—and corporate bailouts specifically—has been largely negative in light of the moral-hazard problems caused by such interventions. Even in the banking literature, where a few exceptions (discussed below) can be found, bailouts rarely garner more than grudging approval. This academic skepticism has a long pedigree. As early as 1873, Walter Bagehot wrote:

If the banks are bad, they will certainly continue bad and will probably become worse if the Government sustains and encourages them. The cardinal maxim is, that any aid to a present bad bank is the surest mode of preventing the establishment of a future good bank.

Almost a century and a half later, Bagehot’s admonition remains popular among economists. In his “Blueprints for a New Global Financial Architecture,” for example, Calomiris (2000) opens with

*Problem 1: Counterproductive financial bailouts of insolvent banks, their creditors, and debtors by governments, often assisted by the IMF, have large social costs.* Bailouts are harmful for several reasons. First, they entail large increases in taxation of average citizens to transfer resources to wealthy risk-takers. Tax increases are always distortionary, and serve to accentuate the unequal wealth distribution. Second, by bailing out risk takers local governments and the IMF subsidize, and hence encourage, risk taking. Moral-hazard incentive problems magnify truly exogenous shocks that confront banking systems. Excessive risk taking by banks results in banking collapses and produces the fiscal insolvency of governments that bail out banks, leading to exchange rate collapse. Banks willingly and knowingly take on more risks—especially default risks and exchange risks—than they would if they were not protected by government safety nets.

And yet, to the consternation of many economists, and contrary to much received academic wisdom, governments have time and again ridden to the rescue of banks and other “too big to fail” institutions during economic crises. Notable recent examples include the controversial bailouts of General Motors and Chrysler in December 2008. Although the federal government provided more than \$80 billion in assistance, both companies had to file for Chapter 11 bankruptcy protection by June 2009. Incumbent shareholders were wiped out, senior management and board members were fired, and labor unions and dealers accepted significant concessions.

Nevertheless, in succeeding years, some commentators have begun to portray the automobile bailouts in more charitable tones. In November 2010, BusinessWeek reported

General Motors Co.’s initial public offering showed that while U.S. President Barack Obama’s administration may lose billions on the auto-industry bailout, the national budget and economy might be better off for it.

The U.S. sold almost half of its stake in the nation's largest automaker for \$33 a share—about \$10 less than it needs to break even. The remaining shares will need to sell for about \$20 higher to make up the difference. GM opened at \$35 and stayed within \$1.11 of that price all day. Selling the remaining shares at that price would produce a loss of about \$9 billion.

That may go down as a bargain. The U.S. would have lost \$28.6 billion in spending on social services and missing tax revenue if not for the bailout of GM, its former lending arm and Chrysler Group LLC, according to a study released Nov. 17 by the Center for Automotive Research in Ann Arbor, Michigan.

“GM ends up an economic contributor to the U.S. economy,...It's manufacturing products, it's creating jobs, it's buying wholesale parts, it's doing what an industrial company is supposed to do.”

Significant academic skepticism remains to this day about the wisdom of bailouts as a categorical matter. Thus, the economic crisis of 2008-9 provides an invitation to explore more systematically when (if at all) bailouts are justified on efficiency grounds, how to structure and finance them, and how to gauge their success.

Our paper takes a first step, proposing a model for analyzing the tradeoffs involved in a governmental bailout of a firm. Unlike many models in the banking literature, our firm's efficiency problems are not the prospect of bank runs, debt overhang, or other issues specific to the financial sector. Rather, we study a more canonical firm, which always operates in the shadow of moral hazard and agency costs. We presume that managers and owners of the firm are interested only in their own private payoffs, indifferent to externalities created by the firm's operations. These externalities accrue to outside constituencies, who derive socially valuable benefits from the firm's activities—benefits the firm can neither capture nor internalize. Such stakeholders include existing and future customers, suppliers, employees, creditors, entrepreneurs, and “communities” that benefit from the firm's operation. Their interests are diffuse and difficult to isolate (much less to coordinate), and they possess little real or formal influence over corporate decisions.

Unlike managers and owners, however, we presume that the government cares about total social welfare. If external stakes are large—and they can be much larger than those of the firm's owner—then the public has a legitimate interest in wanting to keep a moribund firm alive. Yet, this is not easy in our model. The government has no special powers, managerial skills, or access to unique information. It is not an omniscient social planner with the power to micro-manage wages or coerce effort. Rather, its abilities are very limited: It can only provide funds to keep the firm solvent; it can tax firms to finance bailouts; and, it has the power to eliminate existing owners and management of bailout recipients. We do not even assume that the government is capable of committing itself to a future bailout policy.

Within this setting, our model considers when the government can (and cannot) serve a useful purpose in bailing out distressed firms. This is not a simple problem. For all its good intentions, government intervention can exacerbate moral hazard: First, bailouts reduce the incentives of both firms and managers to avoid insolvency. Second, financing a bailout through taxation reduces

profits, which reduces real wages and thus efficient managerial effort. In our model (as well as in practice), government-introduced moral hazard severely limits the government's ability to intervene benevolently. We show that the distortions can be so large that they prevent an efficient bailout even though it would be socially preferable for the firm to continue.

Yet at the same time, moral-hazard concerns do not preclude the possibility of efficient bailouts altogether. To the contrary, we show when bailouts are not only feasible but also welfare enhancing. In this sense, our model can formalize—and in some ways mediate—the debate between proponents of bailouts (who emphasize the social costs of allowing firms to fail), and opponents (who emphasize the moral hazard and efficiency losses of government intervention). Partially agreeing with the former, we argue that bailouts are sometimes both feasible and justified on efficiency grounds. Partially agreeing with the latter, we show that even when it would be socially optimal to keep a firm alive, moral-hazard issues can make the cure worse than the disease.

In addition to characterizing when an efficiency-enhancing bailout exists, we are also interested in its core terms and attributes. As already noted, our government is quite limited. It can tax the profits of some or of all firms; it can impose extraordinary taxes on bailed-out companies that succeed later; it can decide whether to bailout firms and on the amount of the bailout; and, if it does bailout firms, it can fire or keep the incumbent manager, and it can eliminate or retain existing owners. In our model, incumbent owners bring no special expertise to the table that other owners could not provide, so the optimal government bailout either squeezes them out entirely or leaves them with no economic rents going forward. Similarly, the government can elicit more effort from managers prior to financial distress by insisting on punishing (firing) the manager when a bailout is required. When incumbent managers—the ones that have driven the firm into distress—do not have significant firm-specific skills, this improves their *ex-ante* incentives and is thus the optimal government bailout policy.

We further show that financing an optimal bailout in our model is generally best done using a *redistributive* tax. That is, an optimal bailout generally does not impose an extraordinary tax (or its functional equivalent) on rescued firms to contribute to their own bailouts out of future profits. Rather, it taxes healthy firms which are not in need of a bailout, and subsidizes bailout recipients with few/no financial strings attached. This is because the government already fully expropriates existing owners and managers. Taxes levied *ex post* against (new) bailout recipients must in effect be “priced” into the governmental subsidy. And because such taxes inefficiently distort incentives, the government is better off imposing lower extraordinary taxes (and thus paying a smaller bailout subsidy) instead.

As a practical matter for policy makers, our analysis contains three core prescriptions:

1. Governments should be very conservative in rescuing firms, targeting only those that provide large stakeholder externalities and those that have a need for relatively modest capital infusions.

2. Governments should presumptively eliminate existing owners and managers<sup>1</sup> (at least those with only modest or no firm-specific comparative advantages) in bailouts.
3. Governments should finance bailouts through taxation of healthy firms, and not through (extraordinary) taxation on those bailout recipients that later recover.

Our model provides a means for assessing the interventions under the Troubled Asset Relief Program (TARP) in 2008-9. First, Treasury acted prudently when it squeezed out both existing shareholders and incumbent managers of GM and Chrysler. Such actions are consistent with our model—and were well suited to incentivize other firms (such as Ford) to work harder to avoid the same fate. In contrast, in many of the bank bailouts, the government was *not* particularly ambitious in attempting to displace the owners and managers who presided over the banks' declines, much less to claw back managers' accrued salaries and benefits (Bebchuk, Cohen and Spamann (2010)). (It is difficult to argue that the managers and traders that had failed to curb the risks of these firms were so valuable that they were irreplaceable.) Our model suggests that full expropriation, not only of the shareholders and their boards, but also of the managers, should be a presumptive aspect of an optimal bailout as a means for deterring future crises.

Second, our model causes us to be critical of the government's intermittent emphasis on generating an "investment return" from bailout funds. In virtually every TARP bailout, the government functionally levied an extraordinary 'tax' on recipients by structuring its cash infusions as either loans or stock purchases. The political appeal of generating a return from ownership stakes is apparent. However, our model points out that government ownership reduced the participation of private investors, which we assume to have superior ability to run corporations. In turn, this reduced the efficiency of bailed-out firms. The government stakes may therefore have crowded out private investors, increased the up-front bailout costs, and diminished the prospects for success. A preferable alternative would have been for the government to sell immediately its stakes to private owners who possessed better capabilities to run the failed firm than the government itself.<sup>2</sup>

Finally, our model suggests that bailouts are better funded through special taxes on healthy firms than through retrospective assessments on bailout recipients (in the form of government owned debt and equity). Early drafts of the 2010 Dodd-Frank Act proposed the creation of \$50 billion bailout fund financed by healthy firms deemed to be systemically significant. Ironically, opponents in the Senate ultimately blocked this proposal, asserting that it would *exacerbate* moral hazard (Schwarcz (2011)). Our model suggests, quite to the contrary, that the bailout fund approach can be well suited

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<sup>1</sup>Flannery (2005) argues that banks should issue reverse convertibles—pre-programmed bailouts. Our paper suggests that such financing would be more effective if it were tied with corporate governance sanctions. Any forced conversion due to poor performance could trigger automatic dismissal of the board and management, reduced management pay (or loss of earlier benefits, i.e., a clawbacks). A super-majority of shareholder votes at a special meeting could override this.

<sup>2</sup>In one sense, perhaps, under the GM and Chrysler bailouts the government arguably enjoyed competence that rivaled that of private owners. Its task force, responsible for the initial stage of the resuscitation, consisted of a team of unusually competent outside private-equity capitalists with business expertise. Moreover, it appears that the task force was not as motivated by a desire to enrich itself, as by a desire for public service (which our model does not allow for). Thus, the government may have followed not the letter, but the spirit of our model's recommendation. And, as our model suggests, by 2010, the government had begun to divest its stake in GM.

to *mitigate* the moral hazard that bailouts inevitably introduce.<sup>3</sup>

As with any theoretical model, our prescriptive insights are subject to caveats and limitations that depend on context. First, government may simply be too wasteful to function effectively as a tax administrator or overseer of rescued firms. This undermines the attractiveness of government interventions *writ large*. However, our model is robust to the introduction of waste at moderate levels, which tend to shrink—but not necessarily eliminate—the contingencies where efficient bailouts are feasible and desirable. Of course, when governmental waste grows prohibitively large relative to the social benefits of bailouts, the efficiency case for intervention disappears altogether.

Second, if the government forces out the owners and managers, equally competent replacements may not be readily available. Yet, because these parties are the same ones that presided over their firms' declines, they are plausibly dispensable—as they were in the aforementioned GM and Chrysler bailouts.<sup>4</sup> That said, our model is robust to introducing an efficiency loss when a competent party would have to be succeeded by a less competent replacement. If this loss is large, the government may or may not choose not to remove incumbent owners or managers of bailed out firms, even if committing to do so would still be optimal *ex ante*. Consequently, just as with government waste, the absence of competent replacements tends to contract (but not necessarily eliminate) the set of contingencies where intervention is socially beneficial.

Third, although we constrain our analysis to situations where outside stakeholders cannot be taxed to help fund bailouts, our model easily permits such an extension. Stakeholder taxes directly reduce the need for corporate taxes, thereby expanding circumstances where intervention is socially beneficial.

Fourth, our analysis presumes that the government's goal is to maximize expected social welfare (subject to budget balancing and general incentive/participation constraints). Although this goal seems defensible on both positive and normative grounds, governmental actors may episodically harbor other objectives—such as maximizing the return on investment in bailed-out firms.<sup>5</sup> However, if government really views itself as a taxpayer-sponsored vulture fund, then governmental action is likely ill-suited to address most social externalities problems.

Finally, even when bailouts are justified on efficiency grounds, they may not win popular support among the most focal (or vocal) constituencies. Indeed, we show that the distribution of costs and benefits from bailouts in our model would likely augur resistance among the Chamber of Commerce,

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<sup>3</sup>It is debatable whether the government will have the capability not to intervene if it will be socially optimal to intervene. Our model suggests that a commitment to expropriate the corporate owners and managers may be more effective than an attempt to limit the pool of funds for bailouts in the future. After all, there was no such fund in place in 2008-9, either, and yet the government did rescue firms.

<sup>4</sup>Recall also that, in our model, we do not expect the government to train managers, to manage the firm itself, or to control the firm in an active fashion. It is often implausible that government can do this better than the free market. In our model, government primarily intervenes with funding, and, if it does so, secondarily with the ability to force the firm to fire the manager.

<sup>5</sup>For example, in its March 2011 report, the TARP Congressional Oversight Panel notes that the US Treasury officials at times tended to alternate inconsistently between articulating social welfarist goals (e.g., employment, economic growth, investment) and recouping a return on investment for taxpayers (Congressional Oversight Panel 2011, page 188).

the Business Roundtable, and (perhaps) labor unions: viewed ex ante, even optimal bailouts leave them worse off. (Of course, the same parties will not fight bailouts in actual distress if they can then avoid complete expropriation.) Naturally, when one factors in the benefit of other stakeholders as well, optimal bailouts remain welfare enhancing.

Our paper now proceeds as follows. Section 1 describes the model setup, characterizes the first-best solution, and formally states the optimal bailout design problem. Section 2 analyzes the constraints, working backwards through the equilibria at each stage. Section 3 characterizes the terms of an optimal bailout. Section 4 illustrates the model solutions graphically. Section 5 discusses the effect of different constraints on the government’s ability to raise funds, and the effect of an ability by the firm’s owners to coerce the government to offer additional funds or leave them with ownership. Section 6 reviews some related literature, and Section 7 concludes.

# 1 Framework and Model

Our model’s main ingredients are as canonical as possible. A firm has a project that has an up-front investment cost, and positive revenues if the project succeeds. The success probability increases with the effort of a manager who dislikes effort but can be motivated with a success-contingent wage. The model’s innovations are that a failing firm has a second opportunity to try to restart the project (at a cost) and that the government can intervene.

## 1.1 Sequential Structure

The model has three discrete time periods,  $t \in \{0, 1, 2\}$ , with no inter-temporal discounting, and three participants: a firm, a manager, and the government. The sequence is as follows:

- Time 0:**
- The government announces a bailout policy  $\langle g, T_1, T_2, FG \rangle$ , the terms of which are described below.
- Time 1:**
- The firm decides whether to invest in a project with initial cost,  $I_1 > 0$ .
  - The project can be a “success,” yielding a gross payoff  $R > 0$ , or a “failure,” yielding gross payoff 0.
  - The probability of success depends on managerial effort. We assume the probability of success is the manager’s effort level  $e_1$ . Thus, the project fails with probability  $(1 - e_1)$ .<sup>6</sup>
  - If the project succeeds, the firm must pay the manager a contingent wage of  $w_1 \geq 0$  and the government a tax of  $T_1 \geq 0$ .<sup>7</sup> The game then ends.

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<sup>6</sup>At the cost of tractability, one could introduce alternative algebraic specifications translating effort into success probability. That said, because “effort” has no natural measuring unit, we lose little or no generality from our current setup, which uses convex effort costs and basic parameter restrictions ( $c > (R + S)$ ) to bound  $0 \leq e_1 \leq 1$ .

<sup>7</sup>As is customary in this literature, we do not allow the manager to purchase the firm and thus circumvent the main incentive problem. The nonnegativity of wages is the functional equivalent of limited liability for the manager. It is easy

- If the project fails, the game transitions to time 2, described next.

- Time 2:**
- The government may readjust the remaining components of policy that are not yet sunk,  $\langle g, T_2, FG \rangle$ . This is tantamount to assuming that the government cannot commit itself to a time-inconsistent policy.
  - The firm has a real option of either abandoning the project or “restarting” it for an additional investment of  $I_2 > 0$ . If the firm restarts, the firm can dismiss or keep the manager,  $FF = \{FIRE, RETAIN\}$ .
  - If the firm restarts, it may use a government-provided cash subsidy (or “bailout” ) of  $g \geq 0$ , which effectively underwrites a portion of the reinvestment costs. In addition, if the firm accepts the subsidy  $g$ , and it has not already fired the manager, the government can force dismissal of the manager,  $FG = \{FIRE, RETAIN\}$ .
  - A “restarted” project proceeds much like in the first period, with the exception that there are no additional restarts. Thus, the restarted project can either succeed (again yielding payoff  $R$ ) or fail (yielding payoff  $0$ ), with the probability of success corresponding to managerial effort at time 2,  $e_2$ .
  - If the restarted project succeeds, the firm must pay the manager wages of  $w_2 \geq 0$  and the government an (extraordinary) tax of  $T_2 \geq 0$ .
  - The game ends, regardless of success or failure.

## 1.2 Additional Assumptions

The manager bears a private cost of providing effort of  $c/2 \cdot e_t^2$ , where  $c > 0$  is assumed “large” relative to other parameters.<sup>8</sup> Thus, single-period managerial utility at time  $t$  is  $m_t = e_t \cdot w_t - c \cdot e_t^2 / 2$ .

In designing its bailout policy, we require the government to break even actuarially. This means that if it chooses to adopt a bailout policy ( $g > 0$ ), then the government must be able to raise sufficient revenues through taxes ( $T_1$  and  $T_2$ ) to finance the bailout. There, government bailouts introduce potential inefficiency through the taxes necessary to underwrite bailouts, because taxes distort managerial and firm incentives.

Finally, we assume that there exists an additional public benefit  $S$ , which embodies the payoff of other stakeholders if the firm succeeds. It is not internalized by either the firm or the manager, and we assume that the recipients of these benefits are beyond the taxation power of the government. (We discuss later how the model changes if they are not.)

In all, our framework contains five exogenous parameters, consisting of investment costs  $I_1$  and  $I_2$ , success-contingent revenues  $R$ , a managerial effort cost parameter  $c$ , and external stakeholder

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to introduce private managerial benefits if the project succeeds. In contrast to wages, private benefits are obtained even without managerial effort. Like wages, they accrue only upon success. The conclusions of our model remain unaltered, because the benefits simply serve to reduce the wage that the firm needs to pay managers.

<sup>8</sup>Specifically, we assume that  $c > (R + S)$ . This is necessary and sufficient to keep the first-best effort below 1.



benefits  $S$ . For notational convenience,  $\Omega \equiv \langle I_1, I_2, R, c, S \rangle$ . Our model has eleven endogenous variables, consisting of the manager's efforts in each period ( $e_1, e_2$ ), wages in each period ( $w_1, w_2$ ), the firm's choice of whether to operate in each period  $(\rho_1, \rho_2) \in \{\text{IN}, \text{OUT}\}^2$ , a bailout amount  $g$ , taxes in each period to finance the bailout ( $T_1, T_2$ ), and the decisions whether to fire the manager, both by the firm  $\text{FF} \in \{\text{FIRE}, \text{RETAIN}\}$  and by the government  $\text{FG} \in \{\text{FIRE}, \text{RETAIN}\}$ . We denote them collectively with the strategy profile  $\Sigma \equiv \langle \Sigma_m, \Sigma_f, \Sigma_G \rangle \equiv \langle \langle e_1, e_2 \rangle, \langle w_1, w_2, \rho_1, \rho_2, \text{FF} \rangle, \langle g, T_1, T_2, \text{FG} \rangle \rangle$ .

### 1.3 The First-Best Solution

We first consider the socially first-best effort if it is socially beneficial for the firm to operate.<sup>9</sup> The expected social welfare in period  $t \in \{1, 2\}$  is

$$sv_t = e_t \cdot (R + S) - \frac{c \cdot e_t^2}{2} - I_t. \quad (1)$$

Accounting for transitions between states, ex-ante total expected social welfare is

$$SV = sv_1(e_1) + (1 - e_1) \cdot sv_2(e_2). \quad (2)$$

The first-best effort levels associated with this problem are therefore

$$e_2^{\text{FB}} = \frac{R + S}{c}, \quad e_1^{\text{FB}} = \left( \frac{R + S}{c} \right) - \frac{1}{2} \cdot \left( \frac{R + S}{c} \right)^2 + \frac{I_2}{c}.$$

The second expression shows that anticipated downstream investment costs  $I_2$  enhance the optimal effort level at time 1. This is because transitions to time 2 grow more socially costly in  $I_2$ . In contrast, the investment at time 1 is effectively sunk by time 2 and does not affect the optimal period-2 effort.

These first-best effort levels yield expected total social welfare of

$$\begin{aligned} SV|_{(e_1^{\text{FB}}, e_2^{\text{FB}})} &= sv_1(e_1^{\text{FB}}) + (1 - e_1^{\text{FB}}) \cdot sv_2(e_2^{\text{FB}}) \\ &= (R + S)^2 \cdot \left[ \frac{(R + S)^2 - 4 \cdot c \cdot (R + S) + 8 \cdot c^2}{8 \cdot c^3} \right] + I_2 \cdot \left[ \frac{I_2}{2 \cdot c} - \frac{(R + S)^2}{2 \cdot c^2} + \frac{(R + S)}{c} \right] - I_1 - I_2. \end{aligned}$$

Due to agency costs and financing constraints, this first-best solution is unattainable. It is only a benchmark for later comparisons.

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<sup>9</sup>Appendix A.1 discusses the necessary and sufficient parameter restrictions for it to be socially optimal for the firm to operate, and for the manager's optimal efforts to be interior. We impose these restrictions on what follows.

## 1.4 Optimal Bailout Design

The optimal non-cooperative bailout design problem is the  $\Sigma \equiv \langle \Sigma_m, \Sigma_f, \Sigma_G \rangle$  that maximizes expected total social value  $SV(\Sigma|\Omega)$ , subject to a number of incentive constraints, participation constraints, and budgetary constraints. Each of these constraints warrants a brief discussion before we formally state the design problem.

The first set of constraints concerns incentive compatibility: Both the manager's and the firm's strategies ( $\Sigma_m$  and  $\Sigma_f$ , respectively) must be part of a Bayesian-perfect equilibrium, so that they choose strategies that maximize their expected payoffs— $M(\Sigma|\Omega)$  and  $\Pi(\Sigma|\Omega)$ , respectively, where  $M$  is managerial utility and  $\Pi$  is corporate profit—at every continuation game, given their beliefs.<sup>10</sup> In addition, we require that the government's strategy  $\Sigma_G$  must also be part of a Bayesian perfect equilibrium. Given that the government is presumed to maximize  $SV(\cdot)$ , this is functionally equivalent to assuming that the government has limited ability to commit and thus maximizes social welfare at each continuation stage. More formally, if  $\Gamma(SV(\cdot), M(\cdot), \Pi(\cdot), \Sigma, \Omega)$  denotes the game established by our framework, and  $B(\Gamma)$  denotes the Bayesian perfect equilibria of  $\Gamma$ , incentive compatibility requires that  $\langle \Sigma_1, \Sigma_2, \Sigma_3 \rangle \in B(\Gamma)$ .

The second set of constraints concerns participation by the firm and the manager. We require that under the optimal bailout plan, the firm is still willing to invest, and the manager is still willing to work for the firm. Thus, we require both parties to achieve expected payoffs that exceed their reservation utilities (which we normalize at zero). Formally, individual rationality requires that both  $M(\Sigma|\Omega) \geq 0$  and  $\Pi(\Sigma|\Omega) \geq 0$ .

The third constraint concerns budgetary feasibility. Specifically, we require that the bailout program achieves actuarial budget balance, so that the expected tax received by the government (through  $T_1$  and  $T_2$ ) can finance the expected bailout costs.<sup>11</sup> In effect, the actuarial budget balance requirement constitutes the government's *de facto* participation constraint. Formally, actuarial budget balance requires that

$$e_1 \cdot T_1 + (1 - e_1) \cdot (e_2 \cdot T_2 - g) \geq 0 .$$

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<sup>10</sup>For the exposition here, we slightly abused notation by not differentiating between the period-2 strategies/payoffs of the retained manager and those of her replacement (if the incumbent is fired). This is without loss of generality, because the incumbent and replacement cases are mutually exclusive, and because they face identical continuation payoffs / strategies as of period 2. That said, we are careful to distinguish in what follows how the retention of the incumbent affects her *first* period behavior.

<sup>11</sup>The assumption that the government *actuarially* balances its budget (instead of on a firm-by-firm basis) is natural. The government regulates an entire population of firms and balance budgeting is a condition that holds in the aggregate. Actuarial budget balancing is only the per-firm analog of an aggregate budget-balance condition.

The optimal bailout design problem can now be stated formally as

$$\max_{\langle \Sigma_m, \Sigma_f, \Sigma_G \rangle} SV(\Sigma|\Omega) \quad (3)$$

$$\begin{aligned} \text{subject to:} \quad & \text{(IC)} : \langle \Sigma_m, \Sigma_f, \Sigma_G \rangle \in B(\Gamma) \\ & \text{(IR)} : M(\Sigma|\Omega) \geq 0; \quad \Pi(\Sigma|\Omega) \geq 0 \\ & \text{(BB)} : e_1 \cdot T_1 + (1 - e_1) \cdot (e_2 \cdot T_2 - g) \geq 0 \end{aligned}$$

where (IC), (IR), and (BB) are the incentive, participation, and budget-balance constraints.

## 2 Analysis of Constraints

In order to solve for the optimal bailout, it is necessary to describe the constraints within each continuation game, working backwards in sequence. We begin with the restart stage, then move back to the initial project stage, and finally describe the optimal design of bailout policy.

### 2.1 Restarted Projects (Period 2)

Consider the players' optimal strategy at time 2 for a project that has failed at time 1, supposing first that the firm has restarted the project (so that investment  $I_2$  and bailout  $g$  are already sunk). The firm offers the manager a contract paying  $w_2$  in the event that the project succeeds at time 2, and zero otherwise. (The continuation game is the same if the period-1 manager is replaced.) The manager now expects to receive  $m_2 \equiv e_2 \cdot w_2 - c \cdot e_2^2/2$ . Her payoff-maximizing effort is  $e_2^*(w_2) = w_2/c$ , which results in expected continuation profits for the firm of  $\pi_2 = e_2^*(w_2) \cdot (R - T_2 - w_2) - I_2$ . Assuming the firm operates, it optimally pays a success-contingent wage of  $w_2^* = (R - T_2)/2$ . This induces a managerial effort level of<sup>12</sup>

$$e_2^* = \frac{R - T_2}{2 \cdot c}.$$

The continuation value of a restarted project for the manager is therefore

$$m_2^*(g, T_2) = e_2^* \cdot w_2^* - \frac{c \cdot e_2^{*2}}{2} = \frac{(R - T_2)^2}{8 \cdot c}.$$

The firm's expected time 2 profits (inclusive of bailout subvention and restart costs) are

$$\pi_2^*(g, T_2) = e_2^* \cdot (R - T_2 - w_2^*) = \frac{(R - T_2)^2}{4 \cdot c} + g - I_2.$$

<sup>12</sup>Even when  $T_2 = 0$ , managerial effort is below first-best,  $e_2^{\text{FB}} = (R + S)/c$ . The government has neither the ability to set wages nor the ability to coerce the manager into providing first-best effort. (It can also not pay the firm negative taxes to increase the incentives.) Thus, governmental intervention is not able to achieve first-best.

Sequential rationality requires that the firm restarts if and only if its expected profits are nonnegative. The firm's optimal restart strategy is

$$\rho_2^* = \begin{cases} \text{IN} & \text{if } \pi_2^*(g, T_2) \geq 0, \\ \text{OUT} & \text{otherwise.} \end{cases}$$

Larger governmental subsidies push the firm toward restarting, while taxes on successful restarts push the firm towards abandoning.

Finally, the government's interim payoff at this stage is

$$sv_2^*(g, T_2) = \begin{cases} \left(\frac{R-T_2}{2 \cdot c}\right) \cdot \left(\frac{3 \cdot R + 4 \cdot S + T_2}{4}\right) - I_2 & \text{if } \rho_2^* = \text{IN} \\ 0 & \text{otherwise.} \end{cases}$$

As long as the firm reinvests,  $sv_2^*(g, T_2)$  is invariant in  $g$ , and strictly decreasing in  $T_2$ . Moreover, Bayesian perfection also requires sequential rationality from the government, so that  $\langle g, T_2, FG \rangle$  must remain optimal at time 2. This leads to the following lemma:

**Lemma 1** *The optimal bailout policy sets  $T_2^* = 0$ . Moreover, the optimal bailout pays a positive subsidy  $g^*$  only if  $R^2/(4 \cdot c) - I_2 < 0$ , and sets  $g^* = I_2 - R^2/(4 \cdot c)$ , so that the firm earns zero continuation rents, i.e.,  $\pi_2^*(g^*, T_2^*) = 0$ .*

(All lemmas are proven in the Appendix.) Although it is surprising at first that  $T_2 = 0$ , the intuition is straightforward. Any marginal increase in  $T_2$  inefficiently reduces managerial effort. As  $T_2$  increases, the tax distortion causes firm profits to fall even faster than tax revenues rise, sharpening the government's balance budget constraint. Consequently, the government can do no better than to drive  $T_2$  to zero, and instead to finance any bailout entirely by time 1 tax revenues (and paying out the residual bailout surplus, if any, as a cash grant to the firm or other members of society). In contrast, although  $T_1$  also distorts first-period effort, it does not necessarily do so in a self-defeating way, because  $T_1$  is not capitalized into the required bailout subsidy.<sup>13</sup>

This insight in turn provides the basis for the second part of Lemma 1. Optimal bailouts must raise revenue solely from  $T_1$ , but the distortions caused by  $T_1 > 0$  are socially costly as well. Any optimal bailout should avoid "overfunding" the minimal subsidy needed to induce continuation. Consequently, an optimal bailout must set both  $g$  and  $T_1$  to the lowest amount that prevents closing. It also implies that the optimal policy does not subsidize firms that would reinvest without government intervention (i.e., those for whom  $\pi_2^*(0, 0) = R^2/(4 \cdot c) - I_2 \geq 0$ ). Therefore, Lemma 1 also states that when the optimal bailout involves a positive transfer, it sets  $g$  to be  $I_2 - R^2/(4 \cdot c)$ , just enough to induce reinvestment but leaving the existing owners with zero continuation profits. (In practice, this would often be done through expropriation in a formal Chapter 11 bankruptcy.)

A direct implication of Lemma 1 is that any feasible bailout funded *entirely* by  $T_2$  (with  $T_1 = 0$ ) is worse than a government non-intervention policy. This is because any bailout that is funded

<sup>13</sup>This result contrasts with the standard "Ramsey pricing" intuition in optimal tax literature, where the taxes on two activities (here time 1 and 2 production) would be set to equilibrate marginal efficiency costs.

entirely by  $T_2$  revenues can only bail out firms that would restart on their own, i.e., without any assistance (or tax) from the government.

Collecting results from the analysis of time 2, interim incentive and participation constraints reduce to the following conditions:

$$w_2^* = \frac{R}{2}, \quad e_2^* = \frac{R}{2 \cdot c}, \quad T_2^* = 0, \quad g^* = \begin{cases} 0 & \text{if } \pi_2^*(0,0) \geq 0 \\ -\pi_2^*(0,0) & \text{if } \pi_2^*(0,0) < 0 \end{cases}, \quad \rho_2^* = \begin{cases} \text{IN} & \text{if } \pi_2^*(g^*, T_2^*) \geq 0 \\ \text{OUT} & \text{otherwise} \end{cases},$$

where  $\pi_2(0,0) = R^2/(4 \cdot c) - I_2$ . We will impose these conditions in the analysis of the time 1 behavior that now follows.

## 2.2 Initial Projects (Period 1)

We now characterize how the firm and manager determine first-period wages and effort, as well as whether the firm invests in the first instance. The players' strategic interactions take place in the shadow of the government's bailout transfer, which can take on one of two values,  $g^* \in \{0, I_2 - R^2/4 \cdot c\}$ . We therefore subdivide our analysis into two cases: (A)  $g^* = 0$ , which is non-intervention; and (B)  $g^* = I_2 - R^2/4 \cdot c$ , where the firm is rescued, but existing shareholders realize zero continuation payoffs.

### 2.2.1 Case A: Non-intervention ( $g^* = 0$ )

If  $g^* = 0$ , the government has no bailout costs and maximal slack in its financing constraint. Thus,  $T_1^* = T_2^* = 0$ . This is complete nonintervention.

If the firm never starts, all profits and wages are zero. If it does start, firms enters the continuation game only with probability  $1 - e_1$ , because with probability  $e_1$  the game has ended with immediate success. If the parties do enter the continuation game, their respective payoffs depend on whether the firm reinvests (which occurs only if  $\pi_2^*(0,0) = R^2/(4 \cdot c) - I_2 \geq 0$ ); and if so, whether the incumbent manager is retained into the second period. We must therefore distinguish between three subcases:

1. The firm abandons the project because  $\pi_2^*(0,0) < 0$ .
2. The firm restarts the project because  $\pi_2^*(0,0) \geq 0$ , and fires the incumbent manager.
3. The firm restarts the project because  $\pi_2^*(0,0) \geq 0$ , and retains the incumbent manager.

**Subcase 1: Firm Abandons Project.** If  $\pi_2^*(0,0) < 0$ , absent governmental intervention, the firm always abandons a failed project.<sup>14</sup> With no chance of continuation, both the firm and manager

<sup>14</sup>This is the only subcase where an efficient bailout *may* exist. We will therefore focus on this case again in the government intervention case in Section 2.2.2.

treat this as a one-period game. Consequently, the manager expects a payoff of

$$M \equiv e_1 \cdot w_1 - \frac{c \cdot e_1^2}{2},$$

which implies managerial effort of  $e_1^*(w_1) = w_1/c$ . Anticipating effort, the firm maximizes

$$\Pi|_{\rho_2=\text{OUT}} \equiv e_1^*(w_1) \cdot (R - w_1) - I_1,$$

which implies a profit-maximizing wage of  $w_1^* = R/2$ . At this wage, the optimal managerial effort is

$$e_1^* = \frac{R}{2 \cdot c}. \quad (4)$$

The firm's maximized profits are

$$\Pi|_{\rho_2=\text{OUT}} = \frac{R^2}{4 \cdot c} - I_1.$$

In sum, when restarts are unprofitable, the firm's optimal first-period investment strategy is

$$\rho_1^* = \begin{cases} \text{IN} & \text{if } \Pi|_{\rho_2=\text{OUT}} \geq 0, \\ \text{OUT} & \text{otherwise.} \end{cases}$$

**Subcase 2: Firm Restarts Project With Different Manager.** Now suppose that  $\pi_2^*(0,0) \geq 0$ , so that the firm restarts, and assume as well that the firm could commit to fire the incumbent manager after an initial failure. Thus, at time 1, the manager continues to view this as a single-period game (at least for her), with payoff of

$$M \equiv e_1 \cdot w_1 - \frac{c \cdot e_1^2}{2}.$$

Consequently, her first-period effort remains  $e_1^*(w_1) = w_1/c$ . In contrast, the firm anticipates operating in the second period under a different manager. Its continuation profit is thus

$$\begin{aligned} \Pi|_{\text{FF}=\text{FIRE}}(e_1) &= e_1 \cdot (R - w_1) + (1 - e_1) \cdot \pi_2(0,0) \\ &= \frac{(R^2 - 4 \cdot c \cdot I_2) \cdot (c - w_1)}{4 \cdot c^2} + \frac{w_1 \cdot (R - w_1)}{c} - I_1, \end{aligned}$$

which implies a profit-maximizing wage of  $w_1^* = R + I_2/2 - R^2/(8 \cdot c)$ . Thus, the manager exerts first-period effort of

$$e_1^* = \frac{R + I_2}{2 \cdot c} - \frac{R^2}{8 \cdot c^2},$$

yielding maximized profits of  $\Pi|_{\text{FF}=\text{FIRE}}(e_1^*)$  (Appendix equation A.1). Note that when the firm can profitably restart, it optimally sets lower-powered first-period incentives than it would in the

absence of the restart option.<sup>15</sup> Intuitively, restarting gives the firm a “second bite at the apple,” which reduces the importance of first-period success. The firm therefore reduces first-period wages (and with it, managerial effort). From the incumbent manager’s perspective, the restart option makes her worse off, because it depresses her first-period wage. The restart benefits accrue to the firm and the replacement manager through the second-period payoffs.

**Subcase 3: Firm Restarts Project With Same Manager.** Finally, suppose once again that  $\pi_2(0,0) \geq 0$ , so that the firm restarts, but now the incumbent manager is retained. In this case, the manager’s payoff is

$$M \equiv e_1 \cdot w_1 - \frac{c \cdot e_1^2}{2} + (1 - e_1) \cdot m_2^* = e_1 \cdot w_1 - \frac{c \cdot e_1^2}{2} + (1 - e_1) \cdot \frac{R^2}{8 \cdot c},$$

which implies effort of  $e_1^*(w_1) = w_1/c - R^2/(8 \cdot c^2)$ . Comparing this expression with the effort in Section 2.2.1 (where the incumbent is fired) shows the moral hazard that managerial retention in a restart induces. If the manager knows that she, too, will receive a second chance if the first-period effort fails, she has an incentive to withhold  $R^2/(8 \cdot c^2)$  first-period effort at every wage level.

Anticipating the effort given wage, the firm maximizes

$$\begin{aligned} \Pi|_{FF=RETAIN} &= e_1^*(w_1) \cdot (R - w_1) + [1 - e_1^*(w_1)] \cdot \pi_2^*(0,0) - I_1 \\ &= \left( \frac{w_1}{c} - \frac{R^2}{8 \cdot c^2} \right) \cdot (R - w_1) + \left( 1 - \frac{w_1}{c} + \frac{R^2}{8 \cdot c^2} \right) \cdot \left( \frac{R^2}{4 \cdot c} - I_2 \right) - I_1, \end{aligned}$$

which implies a profit-maximizing wage of  $w_1^* = (R + I_2)/2 - R^2/(16 \cdot c) > 0$ . Given this wage, the optimal managerial effort is

$$e_1^* = \frac{R + I_2}{2 \cdot c} - \frac{3 \cdot R^2}{16 \cdot c^2},$$

yielding maximized profits of  $\Pi|_{FF=RETAIN}(e_1^*)$  (Appendix equation A.2). Note that first-period wages are higher here than in the case where the incumbent is fired, reflecting the firm’s attempt to dampen managerial moral hazard with higher wages. Nevertheless, the higher wage only helps somewhat—first-period managerial effort remains lower than when the manager is fired.

Comparing subcases 2 and 3 yields the following lemma:

**Lemma 2** *If the incumbent and replacement are equally productive, in the absence of government intervention, the firm should replace the incumbent manager whenever it restarts the project.*

<sup>15</sup>To see this, compare the effort level in the abandonment case to the above expression:

$$e_1^*|_{\rho_2=OUT} = \frac{R}{2 \cdot c} > \frac{R + I_2}{2 \cdot c} - \frac{R^2}{8 \cdot c^2} = e_1^*|_{\rho_2=IN} \Leftrightarrow \underbrace{\frac{R^2}{4 \cdot c} - I_2}_{\pi_2^*(0,0)} > 0.$$

Lemma 2 establishes the optimality of firing the manager after a first-period failure in the absence of government intervention. This is the case even though we do not assume that the firm learns about managerial (lack of) ability from the first-period failure. Intuitively, the credible threat to fire the manager induces greater managerial effort (and less moral hazard) in the first period, because the manager correctly perceives that she has only one chance to succeed. As long as a replacement manager is equally efficient (e.g., faces the same marginal effort costs), then the firm's second period payoffs are the same regardless of whether it is managed by the incumbent manager or her replacement.

Thus, when restarts are profitable and the government does not intervene, the firm always fires the incumbent upon a restart, and the firm's initial investment decision is

$$\rho_1^* = \begin{cases} \text{IN} & \text{if } \Pi_{|\text{FF}=\text{FIRE}}(e_1^*) \geq 0, \\ \text{OUT} & \text{otherwise.} \end{cases}$$

As already mentioned, equal efficiency of managerial replacements sidesteps any time consistency (commitment) problems by the firm (and later the government). Given that the original managers forced the firm into distress, it is plausible to assume that replacement managers are likely to be at least as efficient as the original managers.<sup>16</sup>

### 2.2.2 Case B: Intervention ( $g^* = I_2 - R^2/4 \cdot c$ )

Now consider the case where the government pays a positive bailout with  $g^* = I_2 - R^2/4 \cdot c$ . As previously noted, a necessary (but not sufficient) condition for efficient intervention is  $\pi_2^*(0, 0) = [R^2/(4 \cdot c) - I_2] < 0$ , so that the firm would never restart the project in the absence of the bailout. We therefore confine our discussion below to that parametric case. Although the bailed-out firm reinvests, it earns zero continuation profits, so that  $\pi_2^*(g^*, T_2^*) = 0$ . In the case of intervention, we must distinguish two subcases:

1. The bailed-out firm restarts the project and fires the incumbent manager.
2. The bailed-out firm restarts the project and retains the incumbent manager.

**Subcase 1: Bailed-Out Firm Restarts with Different Manager.** When the incumbent manager knows that she is fired upon a bailout, it is a one-period game for her. Given her wage, she maximizes

$$M \equiv e_1 \cdot w_1 - \frac{c \cdot e_1^2}{2}.$$

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<sup>16</sup>In the real world, if the manager is not fired, it could imply not just that the next-best alternative manager is worse, but that the manager has "captured" the firm's board in the Bebchuk and Fried (2004) sense. If managerial capture causes the firm not to fire the manager, then government intervention can add more value by firing the manager during a restart with a government subsidy.



Her optimal effort is  $e_1^*(w_1) = w_1/c$ . The owners also expect to receive nothing at time 2, and set the wage to maximize

$$\Pi|_{FG=FIRE} = e_1^*(w_1) \cdot (R - w_1 - T_1) + (1 - e_1^*) \cdot \underbrace{\pi_2^*(g^*, T_2^*)}_{=0}.$$

The firm's profit-maximizing wage offer is therefore  $w_1^* = (R - T_1)/2$ . Compare this wage to that in Section 2.2.1, where there was no government and the firm also fired the manager upon the restart. That expression,  $w_1^* = (I_2 + R)/2 - R^2/(8 \cdot c)$ , showed that the continuation investment cost had a positive effect on the wage, while the manager's moral hazard had a negative effect. If the government intervenes and the manager is also fired in such a restart, then the positive impact of the reinvestment cost (+ $I_2$ ) is replaced with the negative impact of first-period taxation ( $-T_1$ ). In addition, the moral-hazard term  $-R^2/(8 \cdot c)$  term is missing. Thus, the incumbent manager may or may not be paid more at time 1 in the presence of a government than in its absence.

Substituting the wage  $w_1^* = (R - T_1)/2$ , the manager's optimal effort level is

$$e_1^* = \frac{R - T_1}{2 \cdot c},$$

and the firm payoffs are

$$\Pi|_{FG=FIRE}(e_1^*) = \frac{(R - T_1)^2}{4 \cdot c} - I_1.$$

**Subcase 2: Bailed-Out Firm Restarts with Same Manager.** If instead the manager is retained, then she expects to receive positive continuation payoffs in the amount  $m_2^*(g^*) = R^2/(8 \cdot c)$ , while the firm continues to expect zero continuation payoffs. Given her wage, the manager maximizes

$$M \equiv e_1 \cdot w_1 - \frac{c \cdot e_1^2}{2} + (1 - e_1) \cdot m_2^*(g^*) = e_1 \cdot w_1 - \frac{c \cdot e_1^2}{2} + (1 - e_1) \cdot \frac{R^2}{8 \cdot c},$$

which yields an effort level (as a function of wages) of  $e_1^*(w_1) = w_1/c - R^2/(8 \cdot c^2)$ . The second term is the manager's option to continue even when the firm is bailed out—i.e., the manager's moral hazard. Therefore, the firm chooses  $w_1$  to maximize its first-period profits,

$$\Pi|_{FG=RETAIN} = e_1^*(w_1) \cdot (R - w_1 - T_1) + (1 - e_1^*) \cdot \underbrace{\pi_2^*(g^*, T_2^*)}_{=0} = \left[ \frac{w_1}{c} - \frac{R^2}{8 \cdot c^2} \right] \cdot (R - w_1 - T_1),$$

where  $T_1$  is the government tax in the first period. The firm's profit maximizing wage offer is  $w_1^* = (R - T_1)/2 + R^2/(16 \cdot c)$ . Substituting this wage into the manager's optimal effort level yields

$$e_1^* = \frac{R - T_1}{2 \cdot c} - \frac{R^2}{16 \cdot c^2}.$$

Note that, similar to the case of non-intervention, first-period effort is lower when the incumbent manager is retained after a bailout. When the manager expects positive rents even in the bailout

state, she rationally spends less in effort costs to avoid that state. Firm profits are

$$\Pi|_{FG=RETAIN}(e_1^*) = \frac{1}{c} \cdot \left( \frac{R-T_1}{2} - \frac{R^2}{16 \cdot c} \right)^2 - I_1 .$$

Comparing the two subcases leads to the following lemma:

**Lemma 3** *As long as the incumbent and replacement are equally productive, the optimal bailout policy replaces the manager whenever the firm receives a bailout.*

Similar to Lemma 2 in the non-intervention case, Lemma 3 states that retaining the manager is never optimal. A credible threat to fire the manager upon a bailout improves her first-period incentives without affecting second period effort. As long as the replacement manager is as proficient as her predecessor, this results in a net gain.

Consequently, when the optimal bailout calls for intervention, the government always fires the incumbent upon a restart, and the firm's initial investment decision is

$$\rho_1^* = \begin{cases} \text{IN} & \text{if } \Pi|_{FG=FIRE} = \frac{(R-T_1)^2}{4 \cdot c} - I_1 \geq 0, \\ \text{OUT} & \text{otherwise .} \end{cases} .$$

A comparison of the firm's total profits with and without government intervention yields the following lemma:

**Lemma 4** *If  $\pi_2^*(0,0) = \left[ R^2/(4 \cdot c) - I_2 \right] < 0$ , government intervention makes the firm and the incumbent manager worse off. The losses to the firm and the manager are increasing in  $T_1$ .*

Without government intervention, the firm's continuation value is zero in the reinvestment stage if  $\pi_2^*(0,0) = \left[ R^2/(4 \cdot c) - I_2 \right] < 0$ , because the firm will choose not to reinvest. With government intervention, the firm reinvests after receiving a bailout  $g^* = I_2 - R^2/(4 \cdot c)$ , but its continuation value is still zero (see Lemma 1). However, the firm is worse off, because the government finances the bailout with taxes  $T_1$  in the initial stage, reducing the firm's total profits. (In response, the firm pays the manager a lower wage, making her worse off as well). Hence, another cost of government intervention is that it increases the tax burden on successful firms (and managers) without an accompanying benefit in the event of a bailout, thereby dampening the incentive for the firm to invest in the initial stage.

### 3 The Optimal Bailout (Period 0)

We can now state our main result. The optimal government bailout policy, with respect to the program in (3), is:

**Proposition 1** *If and only if*

1.  $\frac{R^2}{4 \cdot c} - I_2 < 0$ ;
2.  $I_2 \leq 4 \cdot c - R + \frac{R^2}{4 \cdot c} - \sqrt{16 \cdot c^2 - 8 \cdot c \cdot R}$ ,
3.  $I_1 \leq \left[ R + I_2 - \frac{R^2}{4 \cdot c} + \sqrt{\left( R + I_2 - \frac{R^2}{4 \cdot c} \right)^2 + 8 \cdot c \cdot \left( \frac{R^2}{4 \cdot c} - I_2 \right)} \right]^2 / (16 \cdot c)$ , and
4.  $S \geq \underline{S}$ ,

where  $\underline{S}$  is expressed in Appendix equation (A.6), then the optimal bailout policy is

$$\begin{aligned}
 g^* &= I_2 - \frac{R^2}{4 \cdot c}, \\
 T_1^* &= \left[ R + \frac{R^2}{4 \cdot c} - I_2 - \sqrt{8 \cdot \left( \frac{R^2}{4 \cdot c} - I_2 \right) \cdot c + \left( R - \frac{R^2}{4 \cdot c} + I_2 \right)^2} \right] / 2, \\
 T_2^* &= 0, \\
 FG &= FIRE.
 \end{aligned}$$

Otherwise, the optimal bailout policy mandates non-intervention,  $\langle g^*, T_1^*, T_2^*, FG \rangle = \langle 0, 0, 0, . \rangle$ .

The proof of Proposition 1 is in Appendix A.6. The first condition insures that healthy firms are ineligible for bailouts: i.e., if the firm would restart without government intervention, it should not receive a bailout. The second condition insures that the reinvestment costs  $I_2$  are not so high as to make it impossible to generate enough tax revenue through  $T_1$ . The third condition insures that the firm is still willing to invest in the project under the optimal bailout at time 1. The fourth condition insures that the social benefits  $S$  must achieve some minimal value to justify the intervention in the first place.

The economic intuition for this proposition is as follows. At core, the key efficiency consideration is keeping the manager's effort levels high in both periods. The government can do this only indirectly, by paying the owners a lump-sum bailout. However, such a transfer payment to owners is a blunt instrument—much of it will not be used to incentivize managers, but will be effectively pocketed as profits. (Direct payment to managers is not unhelpful unless it is linked to success—and, if the government were to pay directly to managers, the firm could substitute out its own wage

payments.) Because government subsidies to the firm have to be financed with distortionary taxes, the government is better abstaining from intervention if the firm would restart without a subsidy.

When a positive bailout is paid,  $g$  is set just high enough to keep the firm in business (in order to preserve the social benefits  $S$ ). If  $g$  were any higher than is necessary, taxes would also necessarily be larger (directly disincentivizing effort), and the surplus would go to owners who would not use all of it to incentivize managers to work harder. Thus,  $g$  is the smallest amount necessary to keep the firm in business.

Proposition 1 also restates the intuition (originally from Lemma 1) that the optimal tax structure for financing bailouts sets  $T_2^* = 0$ . In other words, it is *never* optimal to impose an extraordinary tax on rescued firms (or any functional equivalent) requiring them to finance their own bailout from future revenues. Rather, redistributive taxes on the first-period successful firms are the optimal way to finance bailouts.<sup>17</sup> The intuition here is as follows: when the government bails out the firm optimally, it already leaves existing owners with nothing. Any tax  $T_2$  negatively distorts managerial effort, reduces the firm's firm variable profits, and therefore requires an *even larger* transfer payment to restart the firm. (If anything, the government would like to impose a negative tax on revenues, thus rewarding firm owners more if the bailed out firm succeeds. However, we do not allow for success-contingent subsidies.)

This leaves only  $T_1$  to fund any bailout. The tax-revenue it generates depends on managerial effort at time 1, which depends on the managerial wages at time 1, which in turn depends on the tax  $T_1$  that the firm has to pay. The  $T_1^*$  above is the smallest possible self-funded tax that ensures efficient continuation of distressed firms. Even there, however, as Proposition 1 notes, when  $I_2$  is high it may simply be impossible to raise the required from tax revenues. Moreover, even if it were possible to do so, the firm may never afford that opportunity, because it never initially invests. All told, although Proposition 1 makes a case for redistributive bailouts, it also suggests that such bailouts should be utilized only sparingly. Finally, by insisting on the firing of the manager in case of a restart, there will be less need for bailouts, because the managerial moral hazard will be lower.<sup>18</sup>

Analyzing the terms of the optimal tax rate yields the following:

**Corollary 1** *The tax rate  $T_1^*$  financing an optimal non-zero bailout (when it exists) is decreasing in the payoff of a successful project ( $R$ ), increasing in the manager's effort cost ( $c$ ), and increasing in the restart cost ( $I_2$ ).*

As project revenues  $R$  grow, the firm owners become increasingly interested in reinvesting. Consequently, they do not need as much of a subsidy to restart, thereby allowing a lower tax rate. As the

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<sup>17</sup>In successive generations, an ongoing bailed-out firm would presumably pay ordinary time 1 taxes in future periods, just like other firms. Note also that if managers and owners are not fully expropriated in a bailout, it may then become optimal for the government to expropriate some of their residual surplus through other means, such as otherwise non-optimal taxes. This will have positive welfare effects.

<sup>18</sup>Although effort is only one example of moral hazard, our model intuition should readily extend to many situation in which moral hazard creates other distortions (such as excessive risk-taking). By expropriating managers in case of failure, moral hazard is lower.

manager's cost of effort  $c$  and the firm's reinvestment costs grow, restarting becomes less profitable to the firm, thereby necessitating a larger bailout and greater  $T_1$  taxes to finance such.

## 4 Graphical Illustration

We can illustrate the solutions with some examples. We normalize  $S = 1$ , and assume that  $I_1 = 0$  (e.g., the initial firm investment is minimal, or sunk long ago). The investment cost  $I_2 = 0.1$ . Because we will consider revenues from  $R = 0$  to  $R = 1$ , we choose the lowest  $c$  parameter that works for all  $R$ , which is  $c = R + S = 2$ .

Figure 1 is the main figure of our paper. The black dash-dotted line is the first best outcome. Even if the project generates no revenues, the positive externalities mean that it would be socially worthwhile for managers to work. Unfortunately, this would require an unobtainable effort level in the presence of managerial moral hazard that can only be remedied with a success-contingent salary.

For low  $R$ , below 0.3222, it would not be socially optimal to restart the firm if it has failed even in the absence of tax distortions. Irremediable intrafirm moral-hazard inefficiencies are just too strong. Thus, government should not intervene in this region.

For high  $R$ , above 0.8944, it would be privately optimal to restart the firm even without government intervention. The red dashed line is the social value if the original manager is fired upon a restart, the blue line if she is kept on. The red line weakly dominates. Without intervention, at  $R \approx 0.89$ , the expected social value jumps because the firm suddenly chooses to continue.

In the region between 0.3222 and 0.8944, not the firm but only the government would potentially want the firm to continue ( $SV > 0$ ). However, for revenues below 0.7185 if the manager is fired (and 0.7305 if not), the distortionary costs of taxation are too high to make government intervention useful. The graph shows the two Laffer solutions—the inferior solution with a higher tax rate and lower effort sufficient to satisfy the budget constraint is dotted. Thus, for revenues above 0.7185, the government should have a policy of intervention. This gives the social value a jump relative to non-intervention, from a non-intervention value of 0.2764 to an intervention value of 0.3205. (The function is vertical at its vertex.)<sup>19</sup>

Figure 2 illustrates the solution variables associated with these parameters. It plots the tax revenue  $T_1$ , corporate profits  $\Pi$  (including expected continuation values), wages  $w_1$ , managerial effort  $e_1$ , and managerial utility. Our discussion focuses on the cases in which the manager is fired.

Consider the case in which government intervention is just feasible,  $R \approx 0.7185$ .

**Without government intervention**, the firm will shut down. Panel (A) shows that the firm earns profits of  $\Pi \approx 0.065$ . Panel (B) shows that the manager's utility is about  $M \approx 0.032$ . Panel (C) shows that the wage is  $w_1 \approx 0.359$ , and the managerial effort levels are  $e_1 = e_2 \approx 0.180$ .

<sup>19</sup>Note that there is also a kink at 0.89, because the owners' first-order condition changes from optimizing only their first-period profits [with government intervention] to optimizing their two-period profits [because the government now lays off]. Moreover, unlike the government, the government also internalizes the welfare of the manager. This in turn changes all solutions.

**With government intervention**, the firm will restart. Panel (A) shows that a tax of  $T_1 \approx 0.33$  on successful firms raises the funds necessary to pay for bailouts (which are 0.035). Panel (B) shows that the firm now expects to earn a profit of 0.02. This is exclusively from its first-period 10% probability of success, because the government expropriates the existing shareholders in case of a bailout. A world with an active government thus leaves the firm worse off relative to a world without an active government. Panel C shows that the bailout also leaves incumbent managers worse off. They are fired in case of a bailout. This is because, in comparison to the case without a bailout, their wage is lower. However, if we take into account the welfare of the replacement manager (panel D), this is no longer the case. Finally, Panel (F) shows that government intervention lowers the initial in-equilibrium effort of the manager, despite the fact that both owner and manager are expropriated in case of a bailout. This is the unremediable social moral hazard induced by government intervention.

Relative to non-intervention, both the owners and the incumbent managers are worse off. It is only the third party stakeholders that are better off.

Panel (A) shows that, as  $R$  increases, the government can tax less and less. Panel (B) shows that corporate profits approach those in the absence of intervention. Note what happens at the border where the government is indifferent between intervening and not intervening, because the firm is about to continue on without government intervention. Panel B shows that if the manager is not fired, her knowledge of the firm's desire to restart makes her slack off in the first period. This causes a discontinuity for the firm. Interestingly, if the firm could commit itself not to restart so early, it would be better off, because managers would work harder in the first period. (Of course, if the firm could simply fire the manager, as the government does in case of a bailout, then the firm does not face this moral-hazard problem, either.)

## 5 Government Constraints

Our model presumes that the government's budget must be in actuarial balance. That is, the expected tax revenues from first- and second-period production must cover the expected costs of the bailout. Given our result that  $T_2^* = 0$  for any optimal bailout, this means that the expected tax revenues must be collected in the first period. The notion of an actuarial balance is critical, because a single firm cannot both succeed and thus pay taxes, and also require a bailout. This form of actuarial budget balancing is identical to an alternative budget balance condition that requires *aggregate* tax revenues to offset *aggregate* outlays across a population of firms. In essence, the population of "winner" firms (first-period successes) underwrite the bailout population of "loser" firms (first-period failures). Of course, ex ante, because all firms are identical, each is probabilistically both a payer of taxes and a recipient of bailout money.

The isomorphism between actuarial and aggregate budget balance may no longer hold, however, when firms are heterogeneous. (For example, they may face different values of  $R$ .) Firms with high enough  $R$  require no bailouts, because they will finance their own restarts. Firm with low enough  $R$  also receive no bailouts, because it would be inefficient to rescue them. It is only for some firms in

the middle that bailouts are optimal. They may be efficiency justified for some firms but not others. The resulting tax system to finance the bailout can then take numerous forms. One possibility would be to constrain the bailout plan (and its attendant taxes) only to firms who would be “eligible” (on efficiency grounds) for a bailout. Our model can easily be extended to such a framework. Another possibility, would be to tax all firms for first-period success, but target the bailout funds only to those firms for which self financing is unprofitable, and bailouts are socially desirable. Although such a diffuse tax system would impose burdens (with no benefits) on firms that would later not be eligible, it would also spread the marginal inefficiency of the tax across firms, permitting the government to fund the bailout at a lower total inefficiency loss.

Similarly, we did not allow the external stakeholders of the firm—those whose interests are represented by public benefits  $S$ —to be taxed. Employees, contractors, creditors, lessors, customers, and other stakeholders could and likely should in principle help underwrite the costs of a bailout. Of course, in many situations, the stakeholders are too diffuse to be identifiable (e.g., potential future customers, or unspecified “communities” that care about the firm). But even if stakeholder communities were circumscribable, we conjecture that a similar dynamic as in our model would hold—the stakeholders of “winning” firms have to pay taxes to fund bailouts for their counterparts at “losing” firms.

## 5.1 Bailout Recipients, Hold Ups and Lobbying

In our model, the optimal bailout infuses just enough capital to induce reinvestment by the firm, essentially ensuring its continuation profits are zero. It is conceivable that some firms may simply turn down the bailout and refuse to reinvest, holding out for larger transfer payments from the government. This danger may be particularly acute for “too big to fail” firms (e.g., Citibank, Bank of America), where  $S$  is also known to be very high.

We could adapt our model to assume that the bailout recipient firm is able to hold out for some fraction of the total available public surplus,  $sv_2$ . Introducing such hold ups into the model would have negative welfare effects, because it sharpens the government’s budget constraint, and reduces firms’ incentives to avoid bailouts. In turn, such hold ups would require the government to levy a larger first-period tax on successful firms, thereby diluting firms’ incentives to invest *ex ante*. When hold up problems are sufficiently large, bailouts could eventually become unjustified on efficiency grounds and impossible to implement. One possible way to contend with this possibility is to give government the power to *force* a company to accept a bailout (and its attendant terms).

Other forms of influence costs could also be studied in our model, such as the abilities of firms to lobby and distort the government’s process for either structuring the terms of bailout. We conjecture that most of these extensions would similarly tend to reduce the welfare-enhancing effects of targeted bailouts.

## 6 Related Literature

Most of the extant literature on bailouts seems more suitable to bank bailouts, while our own paper seems more suitable to the car company bailouts. Thus, most of our literature review is about papers that focus on banking.

The single closest paper to our own is Philippon and Schnabl (2011), in which a reduction in lending by one bank can reduce other banks' investments. Thus, the benefit of a government bailout is a reduction in the systemic debt overhang, which enhances economically efficient investment. The optimal contract makes each bank pivotal by conditioning a systemic bailout on wide participation.<sup>20</sup> The cost of the bailout is paid for by taxes on household endowments, which causes a parametrically assumed efficiency loss that is linear in the tax required.

Our model shares some features but has different foci. For example, in our model, the social benefits do not accrue to active and taxable participants. Thus, it is not possible to design a contract that makes all parties willing to participate—after all, it is not the firms that lose if a bailout does not take place. (The equivalent of the Philippon and Schnabl (2011) contract would be to make a bailout contingent upon participation of each stakeholder.) Philippon and Schnabl (2011) point out the social externalities among many banks, where all banks benefit from more financial system stability. Our model focuses on bail-outs, in which individual firms can be in trouble, rescuing one does not make it easier or harder to rescue another, and multiple firms need not be rescued at the same time. (Our model can capture some time-varying and cross-sectional variation in the exogenous  $S$  parameter.) Yet another difference of our model from Philippon and Schnabl (2011) is that inefficient investment in our model does not arise because of a debt overhang problem. In fact, we do not view our model as much a model of banks, as we view it of a model of government bailouts for generic non-bank firms, such as the GM and Chrysler. But like Philippon and Schnabl (2011), we focus on the socially inefficient aspects of firm behavior. Bailouts are needed because it is too costly for the firm to finance continuation, given their struggle with their own internal moral-hazard (managerial agency) problems. Our paper focused on the internal and external moral-hazard problems and their best remedies. Our distortive tax effects are endogenous, and not linear as in Philippon and Schnabl (2011). We also offered specific policy recommendations (on managerial and owner retention, bailout funding, etc.) different from those in Philippon and Schnabl (2011), and showed that firms and managers—the Chamber of Commerce—will lobby for a system in which they do not face a priori taxation to cover future bailouts.

Philippon and Skreta (2011) and Tirole (2011) study adverse-selection models, in which firms differ in the publicly-unknown qualities of their existing assets. In contrast, our model studies moral-hazard problems in firms with known assets and opportunities. In their models, the government ends up subsidizing only the worst types, whereas in our model, the government ends up subsidizing

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<sup>20</sup>The paper also shows that if there is excessive participation in the bailout by banks which privately know that they already have *good* projects, then the regulator can improve efficiency by demanding junior securities. It would probably not be difficult to think of a situation in which the problem is too high an uptake by banks which know that they should (socially) not be bailed out. In this case, debt may be a better contract.



only the marginal types. In their models, the parameterized cost of public funds is exogenous, whereas in our model, the costs of raising bailout revenue (through taxes) is endogenous. This is important, because distortionary taxes (and  $T_2$ , in particular) reduce managerial effort and are thus directly responsible for our policy prescriptions. In Philippon and Skreta (2011), the acceptance of government assistance sends a negative signal to outside capital market participants, which increases the recipients' private borrowing costs outside of the program. Philippon and Skreta's (2011) government objective is to obtain a target level of investment that is the cheapest to taxpayers. In our model, the government maximizes total social value instead. Their optimal government intervention is a debt contract. Our's is a direct subsidy (because any government stake raises the required subsidy to entice private participants). In Tirole (2011), there is a spillover aspect of bailouts. The bailout of one firm raises the cost of the next bailout. Firms become progressively more hesitant to participate as progressively weaker banks have already been bailed out before them. Like Philippon and Skreta (2011), the government takes stakes in exchange for its subsidy, though in equity and direct purchases of firm assets, whereas in our model, the government should take no such stake.<sup>21</sup>

There are also pre-2008 crisis papers in the banking literature in which bailouts can be valuable. For example, in Diamond (2001), banks would lose socially valuable information if allowed to disappear. In Gorton and Huang (2004), asset prices depend on liquidity, and the government can enhance asset values by tapping its effectively unlimited credit. In Aghion, Bolton and Fries (1999), managers can be reluctant to liquidate underperforming loans when bank regulators close bankrupt banks aggressively. Bailouts are designed to minimize such welfare losses. The classic paper advocating some government involvement is Diamond and Dybvig (1983), which showed how the private market can fail and how deposit insurance or convertibility restrictions can enhance the social outcome. As expected, deposit insurance immediately raises moral-hazard concerns, e.g., as in Gorton and Rosen (1995), Chan, Greenbaum and Thakor (1992), Boyd and Runkle (1993), Calomiris (1999), etc. *Ex-ante* regulation of the banking sector is usually recommended as a corollary to *ex-post* deposit insurance.

Similarly, in a long number of papers on the subject of government procurement and intervention, Tirole emphasizes how firms can game the system. For example, in Farhi and Tirole (2011), firms correlate on exposures in order to benefit more from government bailouts:

...the central argument of the paper is that private leverage choices depend on the anticipated policy reaction...An accommodating interest rate policy involves (a) an invisible subsidy from consumers to banks (the lower yield on savings transfers resources from consumers to borrowing institutions), (b) current costs, such as the (subsidized) financing of unworthy projects by unconstrained entities, and (c) deferred costs (the sowing of seeds for the next crisis, both through incentives for maturity mismatch, going forward, and the authorities' loss of credibility)...When everyone engages in

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<sup>21</sup>Like us, Tirole suggests "just enough" intervention. Although moral hazard is not central to his analysis, he demonstrates (as do we) that bailouts can exacerbate moral-hazard concerns. The paper does not derive an optimal bailout in the presence of moral hazard.

maturity mismatch, authorities have little choice but intervening, creating both current and deferred (sowing the seeds of the next crisis) social costs.

Finally, Diamond and Rajan (2005) is unusual in that governmental bailouts can backfire even ex-post and themselves cause further insolvencies. Of course, although our paper has not allowed for an additional direct waste parameter in the process of government intervention, we do not suggest that one should ignore the governmental rent-seeking issues first raised in Tullock (1967) and Tirole (1994). Although these papers raise significant real-world problems, our own view of government intervention, though skeptical, is decidedly less pessimistic. Our model's assumption of an optimizing government was a useful analysis device, not a description of real life. To the extent that rent-seeking makes government intervention more costly, it shrinks the regions in which governmental intervention is beneficial. However, the main insight—that when social externalities are large, bailouts can be optimal, and they tend to be better if the managers are dismissed—is robust.

As far as know, no other model of bailouts has focused on the positive *a priori* incentive effects of tying bailouts with managerial dismissals.

## 7 Conclusion

In its final report to Congress in March 2011, the TARP Congressional Oversight Panel (COP) offered the following assessment of the automotive industry bailout:

Treasury's interventions in the automotive industry, in particular, raise moral hazard concerns. In some ways, Treasury actually mitigated moral hazard through its very strict approach to these companies: it forced GM and Chrysler to enter bankruptcy, a step not required of other major TARP-recipient institutions. However, the mere fact that Treasury intervened in the automotive industry, rescuing companies that were not banks and were not particularly interconnected within the financial system, extended the "too big to fail" guarantee and its associated moral hazard to non-financial firms. The implication may seem to be that any company in America can receive a government backstop, so long as its collapse would cost enough jobs or deal enough economic damage. (Congressional Oversight Panel (2011), at 185).

Our model was designed to help frame and assess these concerns. It extended a canonical agency model to explore the trade-offs of governmental interventions when tax-financed bailouts—even as they help third-party stakeholders—could indeed exacerbate moral-hazard problems in firms.

There are three key policy insights that emerged from our model. First, although bailouts can be both feasible and efficiency enhancing, and even for firms outside the financial sector, the government should be cognizant of its negative effects and use bailouts sparingly, rescuing only those distressed firms where external stakes are significant and bailout costs are modest. Second, the government should expropriate existing owners and management in any bailouts in order to

reduce the moral hazard that such bailouts create. And third, the government should not finance bailouts through extraordinary levies on bailed-out firms, but through levies on healthy firms.

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## A Technical Appendix

### A.1 Necessary and Sufficient Conditions For an Interior First-Best Solution

This appendix describes the necessary and sufficient conditions for  $e_1^{\text{FB}}$  and  $e_2^{\text{FB}}$  to be interior.

For  $e_2^{\text{FB}}$  to be interior, there are two conditions:

1. Expected social welfare in the time-2 continuation game must be weakly positive,

$$sv_2(e_2^{\text{FB}}) = \left[ \frac{(R+S)^2}{2 \cdot c} - I_2 \right] \geq 0 \quad \Leftrightarrow \quad I_2 \leq \frac{(R+S)^2}{2 \cdot c} .$$

2. The first-order condition cannot induce an effort level exceeding 1,

$$\frac{(R+S)}{c} \leq 1 \quad \Leftrightarrow \quad c \geq R+S .$$

For  $e_1^{\text{FB}}$  to be interior, there are two binding conditions:

1. It must be that

$$\frac{R+S}{c} - \frac{(R+S)^2}{2 \cdot c^2} + \frac{I_2}{c} \geq 0 \quad \Leftrightarrow \quad I_2 \geq \frac{(R+S)^2}{2 \cdot c} - (R+S) .$$

In other words, if it is too cheap to restart 2 to expend effort, it may not be socially optimal to expend any effort in the first period (other than the requisite start up costs).  $I_2 \geq 0$  guarantees that  $c > R+S$ .

2. Ex-ante, the project must obtain non-negative net social value, or  $SV(e_1^{FB}, e_2^{FB}) \geq 0$ . Using the expression for  $SV(e_1^{FB}, e_2^{FB})$  above, under the first-best effort, it is optimal to invest initially in the project iff

$$I_1 + I_2 \leq (R+S)^2 \left[ \frac{(R+S)^2}{8 \cdot c^3} - \frac{R+S}{2 \cdot c^2} + \frac{1}{c} \right] + I_2 \cdot \left[ \frac{I_2}{2 \cdot c} - \frac{(R+S)^2}{2 \cdot c^2} + \frac{R+S}{c} \right].$$

For  $e_1^{FB} \leq 1$ , it must also be the case that

$$\frac{(R+S)}{c} - \frac{(R+S)^2}{2 \cdot c^2} + \frac{I_2}{c} \leq 1 \quad \Leftrightarrow \quad I_2 \leq c - (R+S) + \frac{(R+S)^2}{2 \cdot c}.$$

However, this constraint is never binding, because the constraints on  $e_2^{FB}$  posit that  $I_2 \leq (R+S)^2/(2 \cdot c)$  and  $c \geq (R+S)$ .

In sum, an interior first-best solution obtains if

$$\begin{aligned} R+S &\leq c, \\ 0 &\leq I_2 \leq \frac{(R+S)^2}{2 \cdot c}, \\ I_1 + I_2 &\leq (R+S)^2 \cdot \left[ \frac{(R+S)^2}{8 \cdot c^3} - \frac{R+S}{2 \cdot c^2} + \frac{1}{c} \right] + I_2 \cdot \left[ \frac{I_2}{2 \cdot c} - \frac{(R+S)^2}{2 \cdot c^2} + \frac{R+S}{c} \right]. \end{aligned}$$

## A.2 Proof of Lemma 1

Note first that a bailout is never optimal when  $\pi_2(0,0) = R^2/(4 \cdot c) - I_2 \geq 0$ , because the firm would refinance the project in the absence of a government bailout, and taxes are purely distortionary. Thus, there is no bailout, and  $g^* = T_1^* = T_2^* = 0$ . The government would consider a bailout only if  $\pi_2(0,0) < 0$ . In Section 2, we showed that for a given  $(g, T_2)$ , the optimal managerial effort in the reinvestment stage is  $e_2^* = (R - T_2)/(2 \cdot c)$  and the firm's continuation value at the reinvestment stage is

$$\pi_2^*(g, T_2) = \frac{(R - T_2)^2}{4 \cdot c} + g - I_2.$$

Thus, the government must choose  $g \geq I_2 - (R - T_2)^2/(4 \cdot c) \equiv g^c > 0$  to make it worthwhile for the firm to continue.

The expected government revenue in the reinvestment stage is  $e_2 \cdot T_2 = (R - T_2) \cdot T_2/(2 \cdot c)$ . Thus, the minimum cost of the bailout, net of government revenue, in the reinvestment stage to encourage the firm to continue is

$$g^c - e_2 \cdot T_2 = I_2 - \frac{(R - T_2)^2}{4 \cdot c} - \frac{(R - T_2) \cdot T_2}{2 \cdot c} = I_2 - \frac{1}{4 \cdot c} \cdot (R^2 - T_2^2),$$

which is increasing in  $T_2$ . Consequently, in the reinvestment stage, the government cannot slacken the budget balancing constraint (BB) by raising  $T_2$ , because this causes firm profits to decline, requiring an even larger transfer  $g$  to encourage the firm to continue.

Furthermore, the social continuation value at the reinvestment stage is

$$sv_2^*(g, T_2) = e_2^* \cdot (R + S) - \frac{c}{2} \cdot e_2^{*2} - I_2 = \left( \frac{R - T_2}{2 \cdot c} \right) \cdot \left( \frac{3 \cdot R + 4 \cdot S + T_2}{4} \right) - I_2,$$

which is decreasing in  $T_2$  but independent of  $g$ . This reflects the fact that the tax  $T_2$  is distortionary, but the transfer  $g$  is not. Thus, because  $T_2 > 0$  does not slacken the constraint (BB) and  $T_2 = 0$  maximizes the social continuation value at the reinvestment stage, it follows that the government optimally sets  $T_2^* = 0$ .

In the reinvestment stage, the government is indifferent among all  $g \geq I_2 - R^2/(4 \cdot c)$  because  $g$  is a pure transfer. However, if  $T_2^* = 0$  the government's budget balancing condition is

$$e_1 \cdot T_1 \geq (1 - e_1) \cdot g,$$

where  $e_1 = (R - T_1)/(2 \cdot c)$  if the initial manager is replaced in the reinvestment stage and  $e_1 = (R - T_1)/(2 \cdot c) - R^2/(16 \cdot c^2)$  if the initial manager is retained. In either case, initial effort is independent of  $g$  and decreasing in  $T_1$ . Thus, it is optimal to set  $g^* = I_2 - R^2/(4 \cdot c)$ .

### A.3 Proof of Lemma 2

From the analysis in Section 2.2.1, we can derive

$$\begin{aligned} \Pi_{\text{FIRE}} &= e_1 \cdot (R - w_1) + (1 - e_1) \cdot \pi_2(0, 0) \\ &= \left[ \frac{(R + I_2)}{2 \cdot c} - \frac{R^2}{8 \cdot c^2} \right] \cdot \left[ \frac{(R - I_2)}{2} + \frac{R^2}{8 \cdot c} \right] + \left[ 1 - \frac{(R + I_2)}{2 \cdot c} + \frac{R^2}{8 \cdot c^2} \right] \cdot \pi_2(0, 0), \end{aligned} \quad (\text{A.1})$$

where  $\pi_2(0, 0) = R^2/(4 \cdot c) - I_2 \geq 0$ , by assumption. From the analysis in Section 2.2.1,

$$\begin{aligned} \Pi_{\text{RETAIN}} &= e_1 \cdot (R - w_1) + (1 - e_1) \cdot \pi_2(0, 0) \\ &= \left[ \frac{(R + I_2)}{2 \cdot c} - \frac{3 \cdot R^2}{16 \cdot c^2} \right] \cdot \left[ \frac{(R - I_2)}{2} + \frac{R^2}{16 \cdot c} \right] + \left[ 1 - \frac{(R + I_2)}{2 \cdot c} + \frac{3 \cdot R^2}{16 \cdot c^2} \right] \cdot \pi_2(0, 0). \end{aligned} \quad (\text{A.2})$$

Hence,

$$\Pi_{\text{FIRE}} - \Pi_{\text{RETAIN}} = \frac{R^2}{256 \cdot c^3} \cdot \left[ 16 \cdot c \cdot (R + I_2) - 5 \cdot R^2 \right] > 0,$$

because  $c > R$ .

### A.4 Proof of Lemma 3

If the replacement manager has the same effort cost as the initial manager, then the social continuation value at the reinvestment stage is identical with either manager and is

$$sv_2^*(g^*, T_2^*) = e_2^* \cdot (R + S) - \frac{c}{2} \cdot e_2^{*2} - I_2 = \left( \frac{R}{2 \cdot c} \right) \cdot \left( \frac{3 \cdot R + 4 \cdot S}{4} \right) - I_2,$$

because  $T_2^* = 0$  by 1. The government's objective is to maximize

$$SV = e_1 \cdot (R+S) - \frac{c}{2} \cdot e_1^2 + (1-e_1) \cdot sv_2^*(g^*, T_2^*) - I_1 ,$$

and

$$\begin{aligned} \frac{\partial SV}{\partial e_1} &= (R+S) \cdot (1-e_2^*) - c \cdot e_1^* + \frac{c}{2} \cdot e_2^{*2} + I_2 \\ &> (R+S) \cdot (1/2) - c \cdot \left( \frac{R-T_1}{2 \cdot c} \right) + \frac{c}{2} \cdot e_2^{*2} + I_2 \\ &> 0 , \end{aligned}$$

where the inequality follows from (i)  $e_2^* = R/(2 \cdot c) < 1/2$  because  $c > R$ , and (ii)  $e_1^* = \frac{R-T_1}{2 \cdot c}$  if the initial manager is replaced and  $e_1^* = \frac{R-T_1}{2 \cdot c} - \frac{R^2}{16 \cdot c^2}$  if the initial manager is not replaced. The result follows because social value is increasing in initial managerial effort and  $e_1$  is higher when the initial manager is replaced.

## A.5 Proof of Lemma 4

If  $\pi_2^*(0,0) = I_2 - \frac{R^2}{4 \cdot c} < 0$  and there is no government intervention, we showed that the firm does not reinvest and its maximized profits are

$$\Pi|_{\rho_2=OUT} = \frac{R^2}{4 \cdot c} - I_1 .$$

In this case, the incumbent manager receives the expected payoff  $M = \frac{R^2}{8 \cdot c}$ .

If  $\pi_2^*(0,0) = I_2 - \frac{R^2}{4 \cdot c} < 0$  and there is government intervention, the incumbent manager is fired, and the firm's maximized profits are

$$\Pi|_{FG=FIRE}(e_1^*) = \frac{(R-T_1)^2}{4 \cdot c} - I_1 .$$

In this case, the incumbent manager receives the expected payoff  $M = \frac{(R-T_1)^2}{8 \cdot c}$ . The results follow immediately.

## A.6 Proof of Proposition 1

The first condition in the proposition (which is necessary but not sufficient) follows immediately from Lemma 1, and no bailout is necessary when  $\pi_2^*(0,0) \geq 0$ . The second condition relates to the financibility of the bailout, and requires the following additional intermediate lemma:

**Lemma 5** *The set of feasible government bailouts is*

$$g \leq (4 \cdot c - R) - \frac{\sqrt{64 \cdot c^2 - 32 \cdot c \cdot R}}{2} .$$

The proof of this lemma is as follows. Because  $T_2^* = 0$  from Lemma 1 and the government prefers replacing the initial manager from Lemma 3, it must be the case that  $e_1^* = \frac{R-T_1}{2 \cdot c}$  and the government's budget balance condition (BB) is

$$\begin{aligned} 0 &\leq e_1^* \cdot T_1 - (1 - e_1^*) \cdot g \\ &= \frac{1}{2 \cdot c} \cdot [(R - T_1) \cdot T_1 - (2 \cdot c - R + T_1) \cdot g] \equiv F(T_1). \end{aligned} \quad (\text{A.3})$$

First, note that  $F(0) = \frac{-(2 \cdot c - R) \cdot g}{2 \cdot c} < 0$  and  $F(R) = -g < 0$ . Second, note that  $F_{T_1} = \frac{R - 2 \cdot T_1 - g}{2 \cdot c} \geq 0$  if  $T_1 \leq (R - g)/2$  and  $F_{T_1} = \frac{R - 2 \cdot T_1 - g}{2 \cdot c} \leq 0$  if  $T_1 \geq (R - g)/2$ . Third, note that  $F(T_1)$  is maximized at  $T_1^{\max} = (R - g)/2$  and  $F(T_1^{\max}) = \frac{1}{8 \cdot c} \cdot [(R + G)^2 - 8 \cdot c \cdot g]$ . Thus, there exists a solution to (BB) if and only if  $[(R + g)^2 - 8 \cdot c \cdot g] \geq 0$ , which is equivalent to the condition  $g \leq (4 \cdot c - R) - \frac{\sqrt{64 \cdot c^2 - 32 \cdot c \cdot R}}{2}$  in the Lemma.

As noted earlier, however, the optimal bailout must sets  $g^* = I_2 - \frac{R^2}{4 \cdot c}$ , which, when combined with Lemma 4, implies that the optimal bailout is financible only if

$$I_2 - \frac{R^2}{4 \cdot c} \leq (4 \cdot c - R) - \frac{\sqrt{64 \cdot c^2 - 32 \cdot c \cdot R}}{2}.$$

This yields the second condition stated in the proposition.

Evaluated at the optimal  $g^*$ , the government's budget constraint (which must be binding in the optimal bailout) becomes

$$I_2 = \frac{e_1 \cdot T_1}{1 - e_1} + c \cdot e_2^2 = \frac{e_1 \cdot (R - 2 \cdot c \cdot e_1)}{1 - e_1} + \frac{R^2}{4 \cdot c}.$$

Solving for  $e_1$  yields

$$e_1^* = \frac{R - \frac{R^2}{4 \cdot c} + I_2 + \sqrt{8 \cdot \left(\frac{R^2}{4 \cdot c} - I_2\right) \cdot c + \left(R - \frac{R^2}{4 \cdot c} + I_2\right)^2}}{4 \cdot c}. \quad (\text{A.4})$$

But because incentive compatibility requires also that  $e_1^* = \frac{R - T_1}{2 \cdot c}$ , we can substitute out first-period effort and solve for optimal first-period taxes,

$$T_1^* = \frac{R + \frac{R^2}{4 \cdot c} - I_2 - \sqrt{8 \cdot \left(\frac{R^2}{4 \cdot c} - I_2\right) \cdot c + \left(R - \frac{R^2}{4 \cdot c} + I_2\right)^2}}{2}. \quad (\text{A.5})$$

Under this tax regime, however, it must still be optimal for the firm to invest in the first period, i.e.,

$$I_1 \leq e_1^* \cdot (R - c \cdot e_1^* - T_1^*) = c \cdot e_1^{*2}.$$

This is equivalent to

$$I_1 \leq I_1^* \equiv \frac{\left[ R - \frac{R^2}{4 \cdot c} + I_2 + \sqrt{8 \cdot \left(\frac{R^2}{4 \cdot c} - I_2\right) \cdot c + \left(R - \frac{R^2}{4 \cdot c} + I_2\right)^2} \right]^2}{16 \cdot c}.$$



This is the third condition in the proposition.

Thus far, we have demonstrated conditions that ensure a bailout is feasible, incentive compatible, financible, and potentially welfare enhancing. Although each condition is necessary for a non-zero bailout to exist, they become sufficient if the bailout is welfare enhancing. The fourth condition in the proposition compares whether the optimal non-zero bailout is socially preferable to non-intervention. That is, it must be the case that the social value (SV from eq. 2), with  $e_1 = e_1^* = (R - T_1^*)/(2 \cdot c)$  (from eq. A.4, assuming that the manager is fired) and  $T_1 = T_1^*$  (from eq. A.5, again assuming that the manager is fired), and  $e_2 = e_2^* = R/(2 \cdot c)$  exceeds social value without intervention, i.e.,  $sv_1$  (eq. 1), with  $e_1 = R/(2 \cdot c)$  (from eq. 4). The condition that  $g > 0$  in the proposition assures that the firm would shut down in the absence of intervention. The critical minimum  $S$  for which government intervention is feasible is then

$$\underline{S} = \left( \frac{2 \cdot c}{2 \cdot c - R} \right) \cdot \frac{(1 - e_1^*) \cdot I_2 - e_1^* \cdot \left( R - \frac{3 \cdot R^2}{8 \cdot c} \right) + c \cdot e_1^{*2} / 2}{e_1^*}, \quad (\text{A.6})$$

where  $e_1^*$  is defined in (A.4).

## A.7 Proof of Corollary 1

As defined in (A.3), the budget (which has to be weakly positive according to (BB)) is

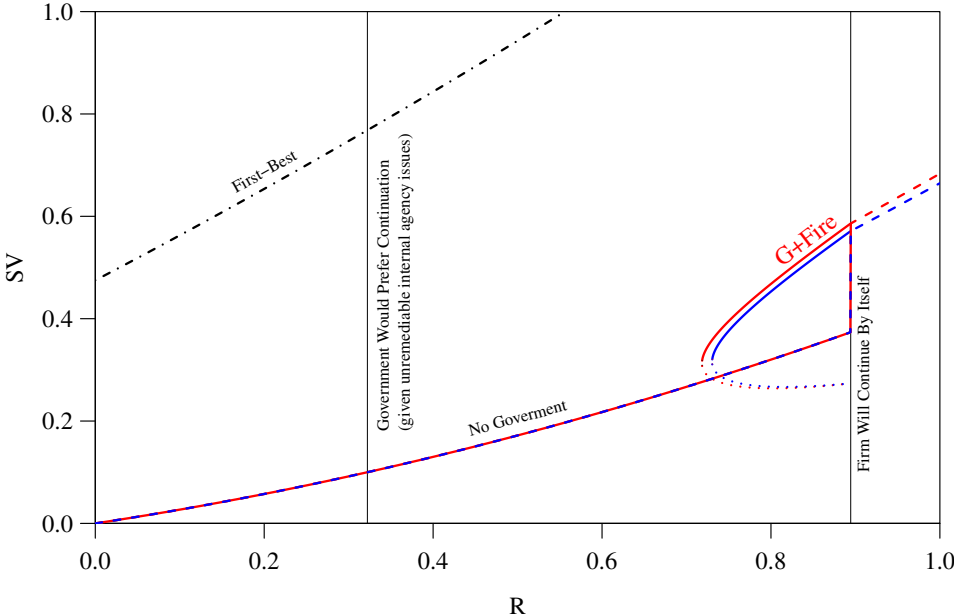
$$F(T) = \frac{1}{2 \cdot c} \cdot \left[ (R - T_1) \cdot T_1 - (2 \cdot c - R + T_1) \cdot g \right].$$

Furthermore, there are two values of  $T_1^*$  such that  $F(T_1^*) = 0$  if  $[(R + g)^2 - 8 \cdot c \cdot g] > 0$ . However, the lower value of  $T_1^*$  is socially desirable, because it leads to higher managerial effort,  $e_1$ . From the proof of Lemma 4, we also know that  $F(T_1)$  first increases in  $T_1$ , then decreases in  $T_1$ . Consequently,  $F_{T_1^*} > 0$  at the lower of the two values of  $T_1^*$ . The comparative-statics results then follow from the Implicit Function Theorem, the fact that  $g^* = I_2 - \frac{R^2}{4 \cdot c}$ , and

$$\begin{aligned} F_R &= T_1 + \left( I_2 - \frac{R^2}{4 \cdot c} \right) + \frac{R}{2 \cdot c} \cdot (2 \cdot c - R + T_1) > 0 \\ F_{I_2} &= -(2 \cdot c - R + T_1) < 0, \text{ and} \\ F_c &= -2 \cdot \left( I_2 - \frac{R^2}{4 \cdot c} \right) - \frac{R^2}{4 \cdot c^2} \cdot (2 \cdot c - R + T_1) < 0. \end{aligned}$$

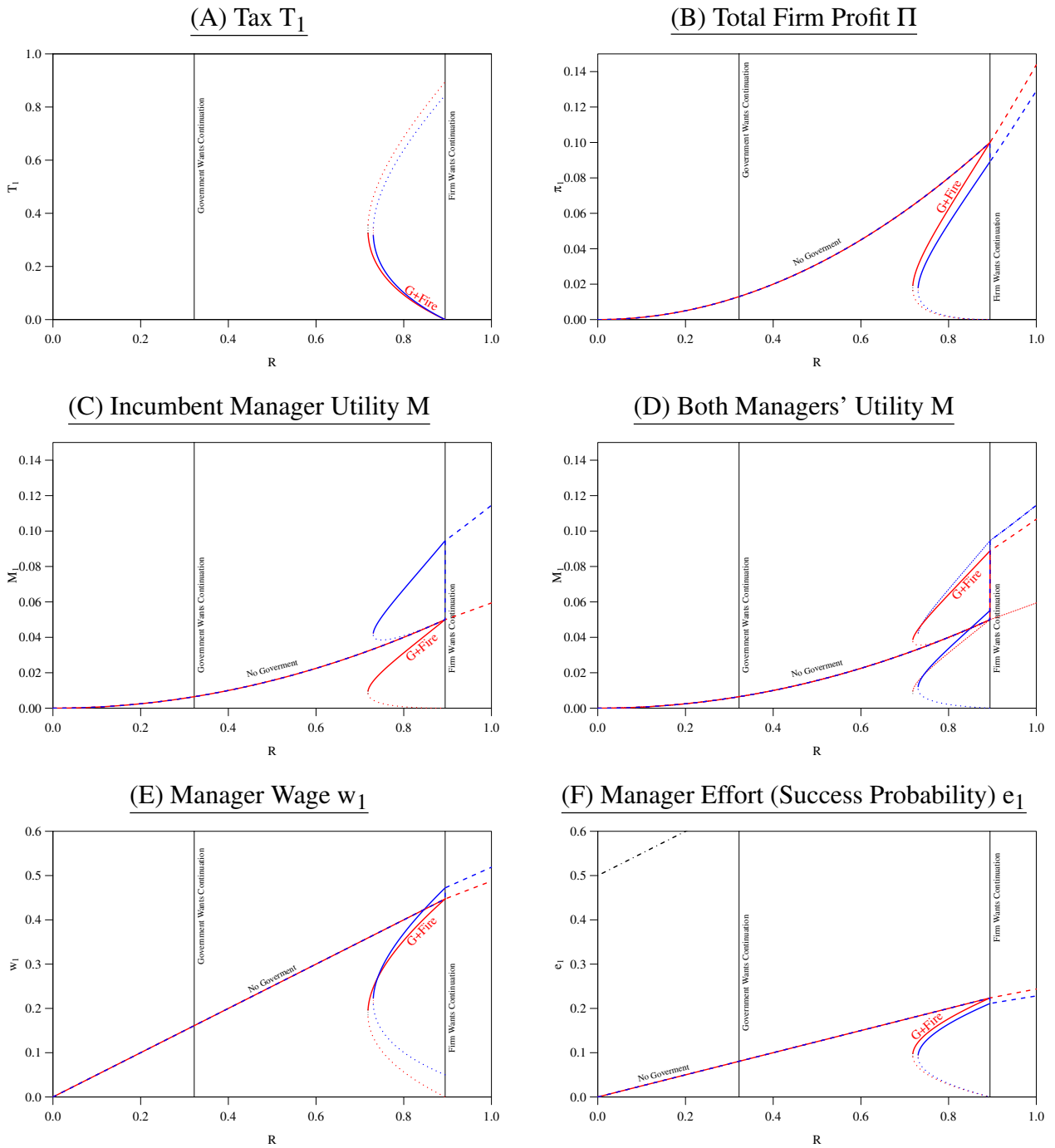
(The optimal tax and intervention are not functions of social surplus  $S$  if the government bails out the firm. It is not valuable for the government to subsidize more than  $g^*$ , regardless of how large  $S$  is.)

Figure 1: The Social Value Function (SV)



**Parameters:**  $S = 1$ ,  $c = 2$ ,  $I_1 = 0$ ,  $I_2 = 0.1$ . Dashed lines apply in the absence of government. Solid lines apply in the presence of government. (Their dotted continuation lines are the inferior Laffer solution.) Blue lines apply in cases in which the manager is retained, red lines to cases in which the manager is fired.

Figure 2: Equilibrium Behavior and Utility as Function of R



**Parameters:**  $S = 1$ ,  $c = 2$ ,  $I_1 = 0$ ,  $I_2 = 0.1$ . Dashed lines apply in the absence of government, except in panel D where additional dashed lines to the left of 0.7185 copy the incumbent manager's utility from panel C. Solid lines apply in the presence of government. (Their dotted continuation lines are the inferior Laffer solution.) Blue lines apply in cases in which the manager is retained, red lines to cases in which the manager is fired. The black dash-dotted line in (F) is the unachievable first-best (effort).